Lecture at the 40th International School of Hydraulics

"Advances in Hydraulic Research"

Multiphase flow modeling using Smoothed Particle Hydrodynamics

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Layout:

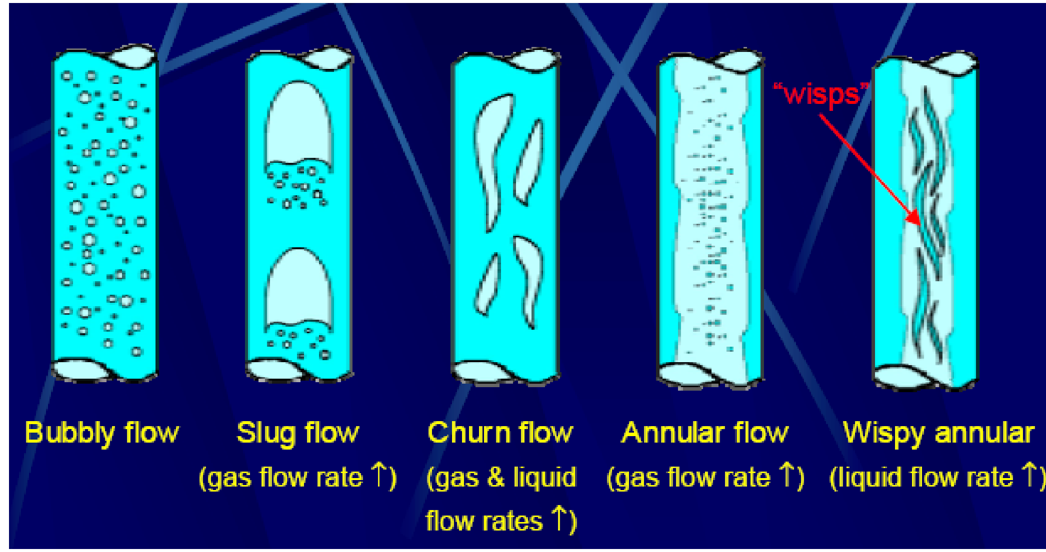
- motivation; SPH fundamentals (why and how?)
- single- and two phase interfacial flows
- two-fluid modeling of dispersed flows

Motivation: engineering (gas-liquid flow structure)

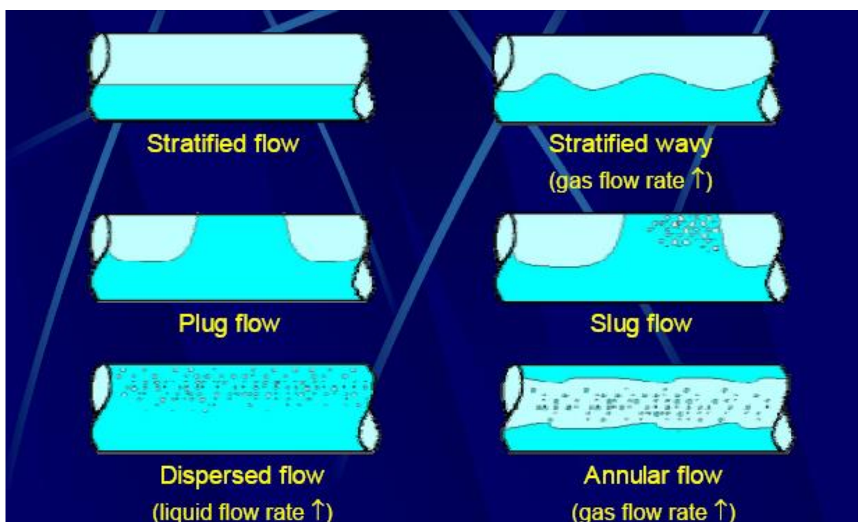
The flow structure,
or regime, depends on:

- flow direction w.r.t. **g**
- mass flow rates of the phases, or superificial velocities
- void fraction

Eötvös number $$Eo = \frac{\Delta\rho g d^2}{\sigma}$$



Vapour-liquid flow structures
in a horizontal/vertical pipe [Bertola 2004]



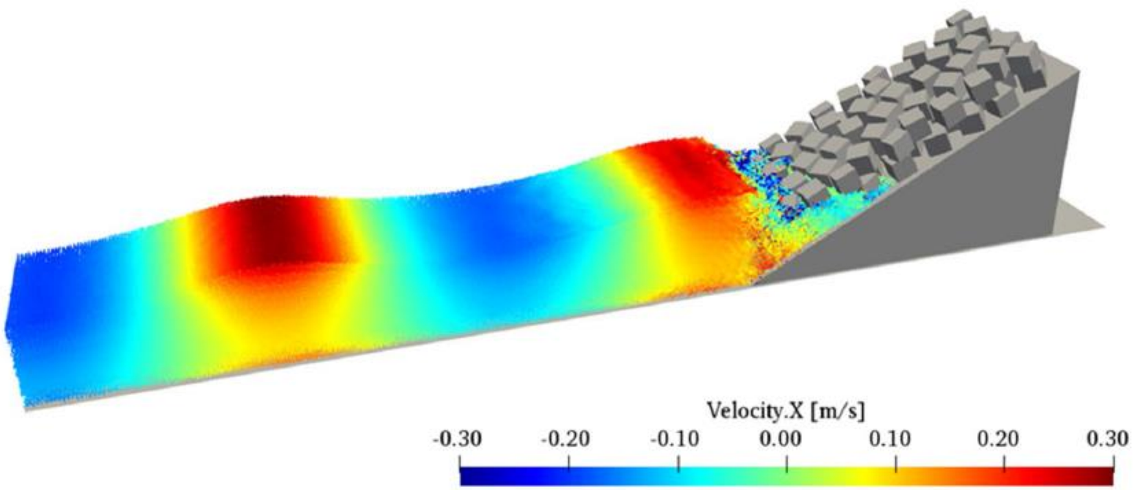
Computational variants:

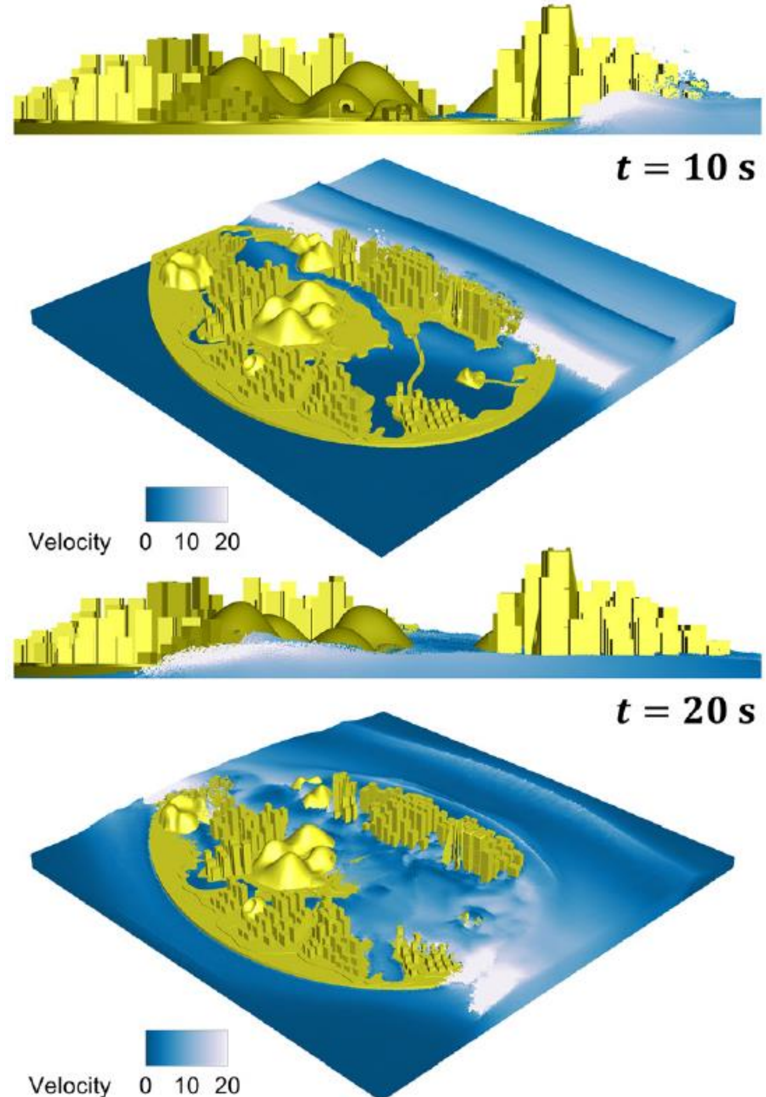
- details of microstructure,
- averaged description

Motivation: coastal engineering (breakwater or … tsunami wave)

SPHydro simulation: tsunami raid over a coastal city [Lyu et al., Phys. Fluids 2023]

DualSPHysics simulation of breakwater flow [Zhang, Crespo, Altomare, J. Hydrodyn. 2018]





Why SPH?

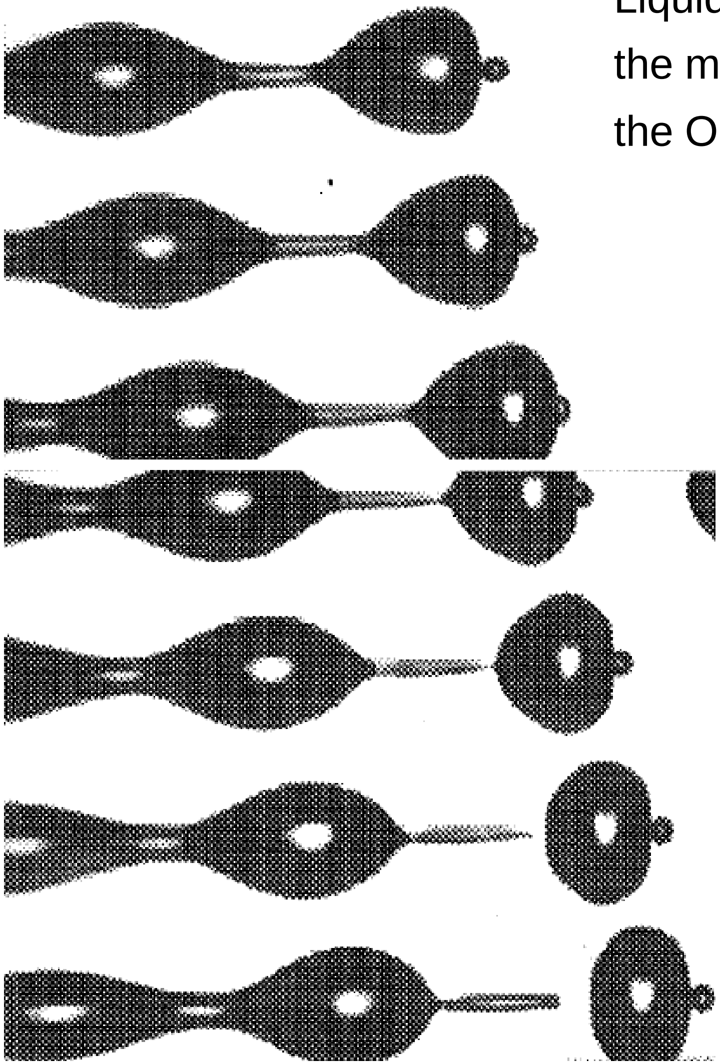
- relatively straighforward input of domain geometry
- free-surface flow w/o mesh

Capillary instability (Rayleigh-Plateau)

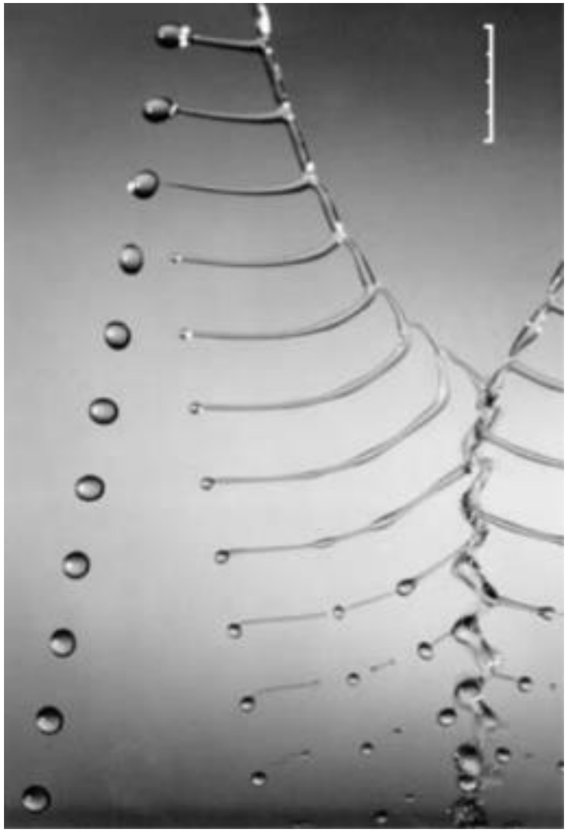
Liquid column fragmentation

the most unstable mode $\lambda=2\pi r\,[2+(18\ \mathrm{Oh})^{½}]^{½}$

the Ohnesorge number $\mathrm{Oh}=\rho v^2/\sigma r$



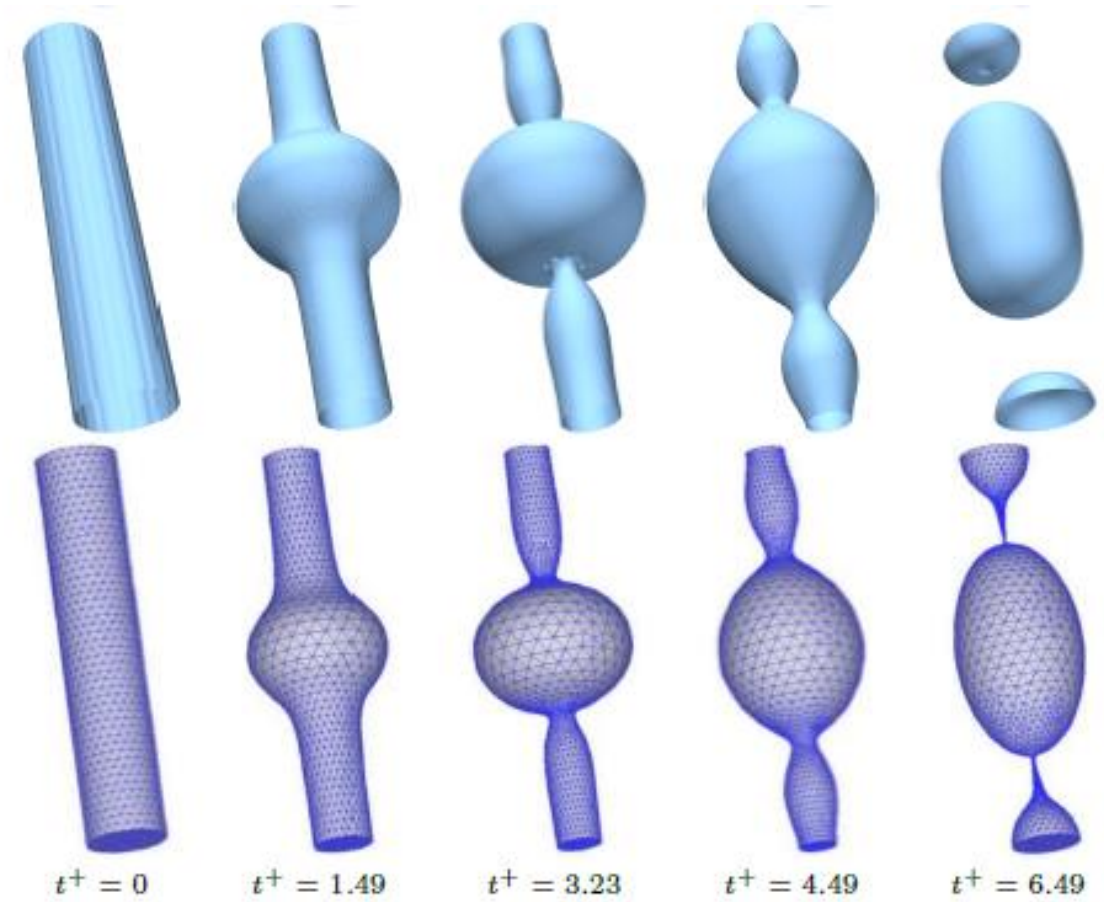
Breakup of a jet issued from ~0.2 mm orifice.
Stroboscopic imaging at ~20μs intervals.
[Kowalewski, *Fluid. Dyn. Res*. 1996]



Oblique collision of two laminar jets:
fishbone instability, a close-up (1cm scale-bar)
[Hasha & Bush, *J. Fluid Mech*. 2004]

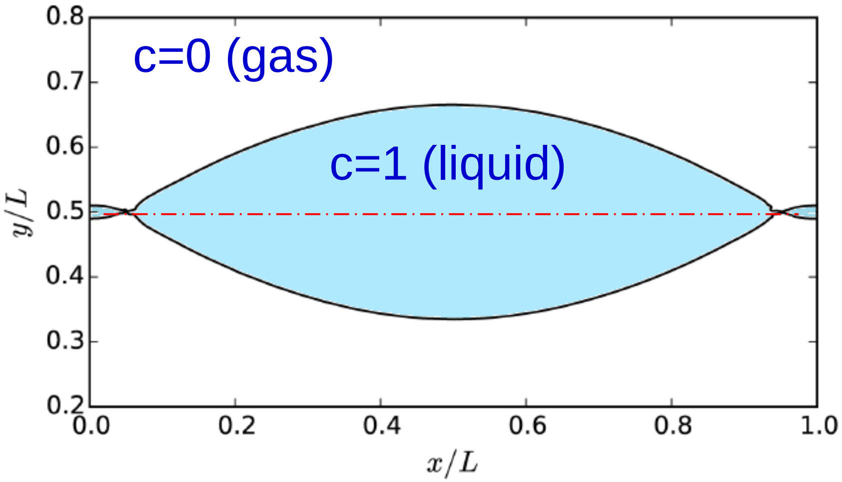
Example of SPH application to multiphase flows

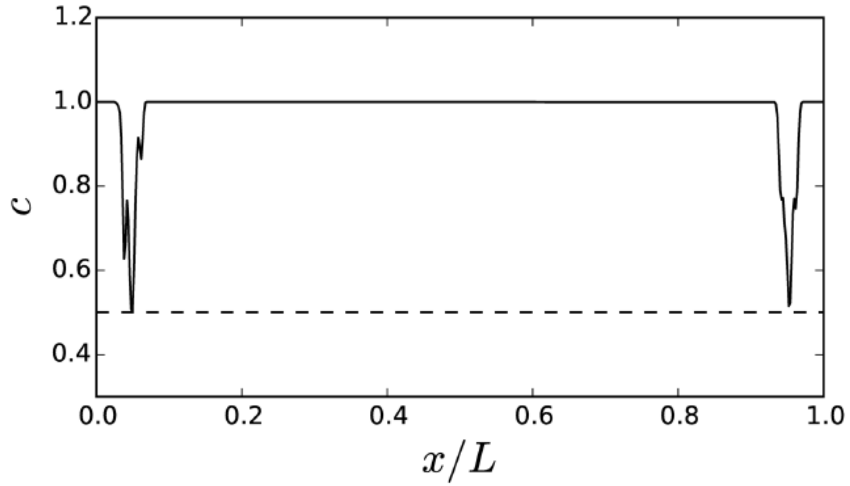
Liquid column fragmentation – the Rayleigh-Plateau instability



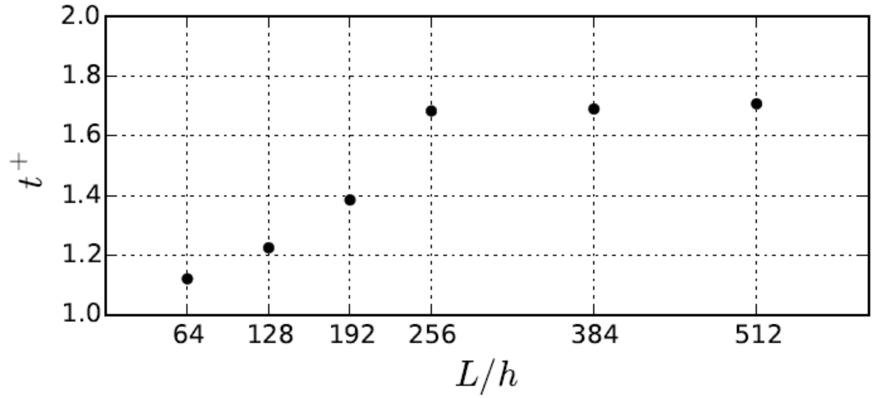
SPH simulation results (top) [Olejnik & Szewc, 2018]
exact fractional step method (bottom) [Dai & Schmidt, 2005]

The Rayleigh-Plateau instability (cntd.)





Determination of the liquid column breakup time:
the profile of phase indicator (colour function *c*)
along the CL (dot-dashed, red).



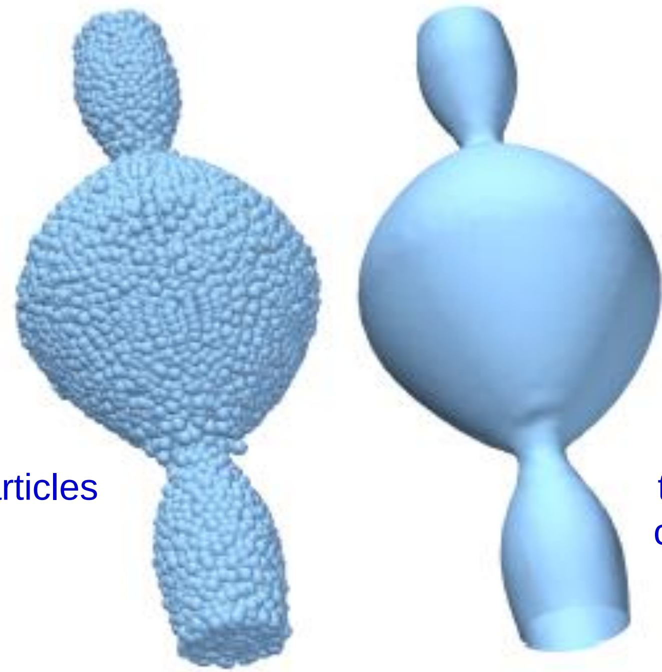
SPH resolution impact: the breakup time.

[Olejnik & Szewc; J. Theor. Appl. Mech. 2018]

What are these coloured particles?
Remark on post-processing and visualisation

- solution of fluid evolution equations in terms of fields

  → post-processing recommended (from particle to field data)



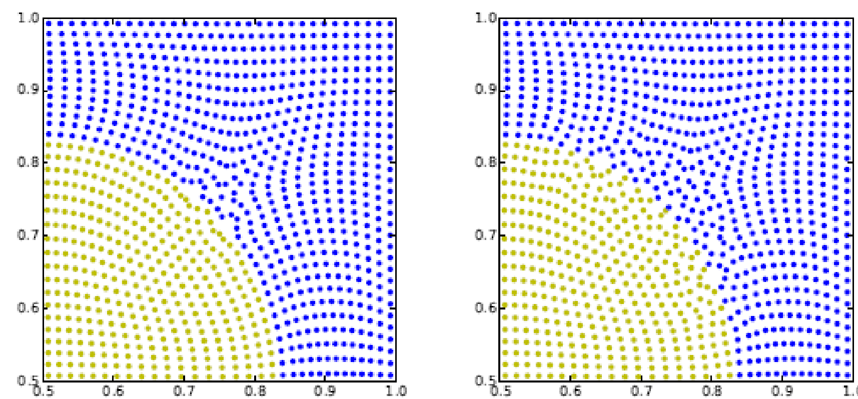
liquid SPH particles
c=1

the interface
c=0.5

[Olejnik PhD, 2019]

- SPH particles are, precisely, interpolation points:

  → they represent some volume (not shape)
  → "voids" are visually troubling



The "squircle" deformation (one quarter shown):
SPH particle positions at two time instants

Making hydrodynamics smooth (using particles)
some historical facts

Smoothed Particle **Hydrodynamics**

- developed for astrophysical simulations:
  [Lucy (1977), Monaghan (1977)]
- alternative name:
  Smoothed Particle Applied Mechanics (SPAM)
  [Nugent & Posch, PRE (2000)]

Smoothed **Particle** Hydrodynamics

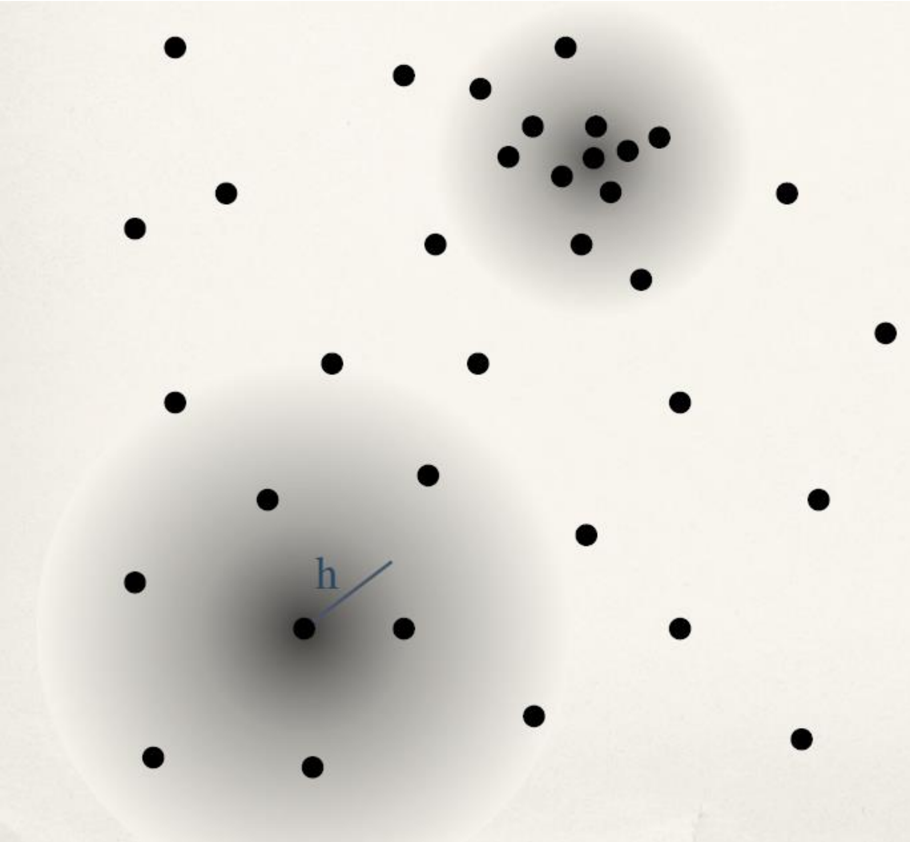
- particle approach:
  convection treated exactly, mass conserved
  [Monaghan, Annu. Rev. Astron. Astrophys. (1992)]

Smoothed Particle Hydrodynamics

- smoothing functions (kernel) used

''resolution follows mass'' →

[Price, SPHERIC keynote (2013)]:



Choice of SPH kernel

properties

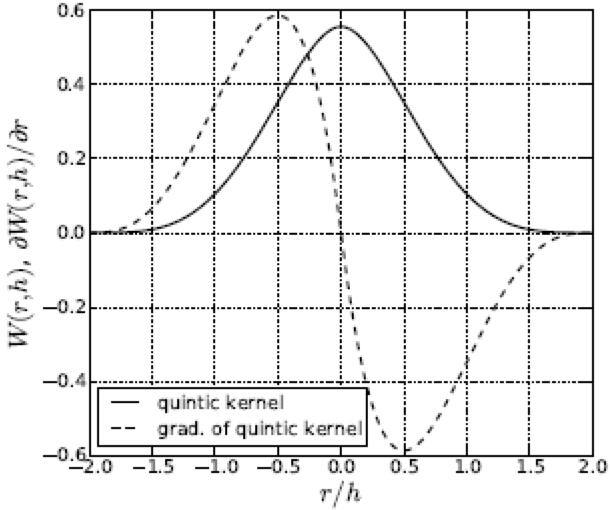
$$W(\mathbf{r},h) = W(-\mathbf{r},h)$$

• necessary properties:
symmetry, limit delta-behaviour, normalisation

$$\lim_{h\to 0} W(\mathbf{r},h) = \delta(\mathbf{r})$$

$$\int_\Omega W(\mathbf{r},h)\mathrm{d}\mathbf{r} = 1$$

• sufficiently smooth function
(to accurately compute derivatives)



• in practice: of compact support
(to limit the number of interacting particles)

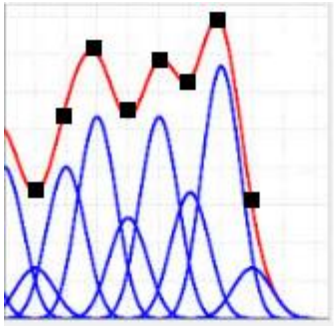
• usually spline polynomials used
(example: cubic B-spline)

$$W(\mathbf{r},h) = \frac{10}{7\pi h^2}\begin{cases} 1-\frac{3}{2}q^2+\frac{3}{4}q^3, & \text{for } 0 \leqslant q < 1 \\ \frac{1}{4}(2-q)^3, & \text{for } 1 \leqslant q < 2 \\ 0, & \text{otherwise,} \end{cases}$$

$$q = |\mathbf{r}|/h$$

Integral interpolation, discretisation

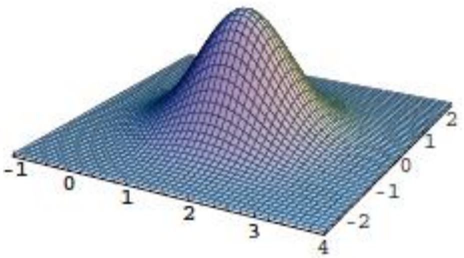
The concepts of integral and summation interpolants:
continuous vs. discrete



$$\hat{A}(\mathbf{r}) = \int_{\Omega} A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}'$$

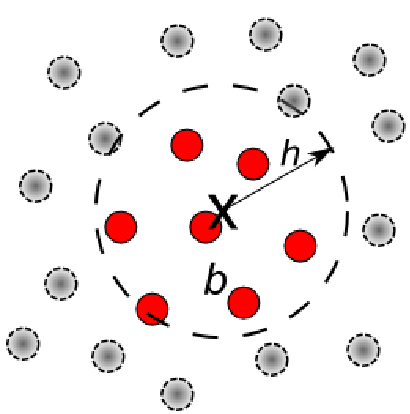
$\hat{A}(\mathbf{r}) = A(\mathbf{r}) + \varepsilon h^2$

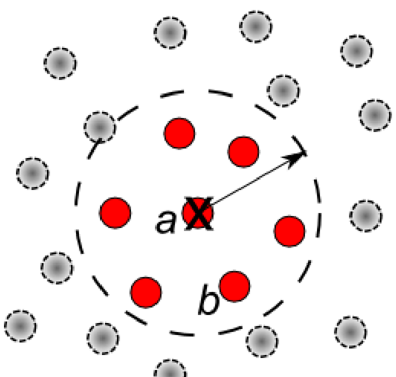
$$\langle A\rangle(\mathbf{r}) = \sum_b A(\mathbf{r}_b)W(\mathbf{r}-\mathbf{r}_b,h)\Omega_b$$



$$d\mathbf{x}' \rightarrow dm/\rho(\mathbf{x}')$$

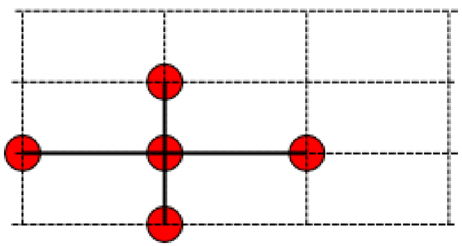
$$\rho_a = \sum_b \rho_b \frac{m_b}{\rho_b} W_{ab} = \sum_b m_b W_{ab}$$

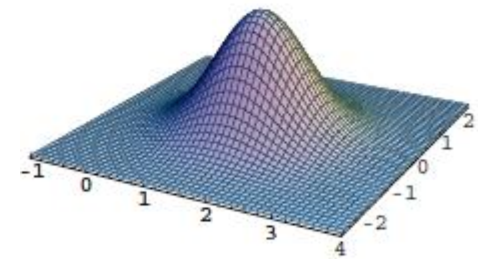




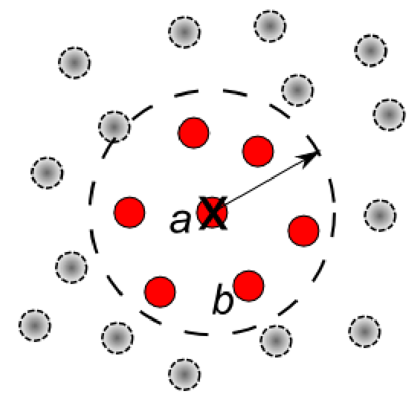
Integral interpolation, discretisation (cntd.)

Representation of differential operators:



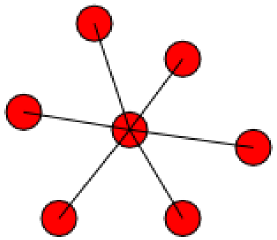


integral interpolant (kernel-based):



$$\widehat{A}(\mathbf{r}) = \int_{\Omega} A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}'$$

$$\widehat{\nabla A}(\mathbf{r}) = \int_{\Omega} \nabla A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}'$$



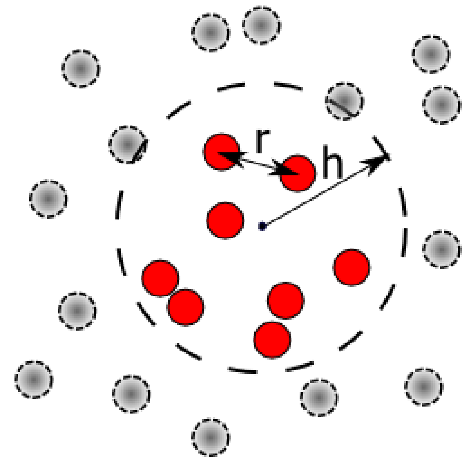
$$\widehat{\nabla A}(\mathrm{r}) = \int_{\Omega} A(\mathrm{r}')\nabla W(\mathrm{r}-\mathrm{r}',h)d\mathrm{r}'$$

$$\langle \nabla A \rangle_a = \sum_b A_b \nabla_a W_{ab}(h)\Omega_b$$

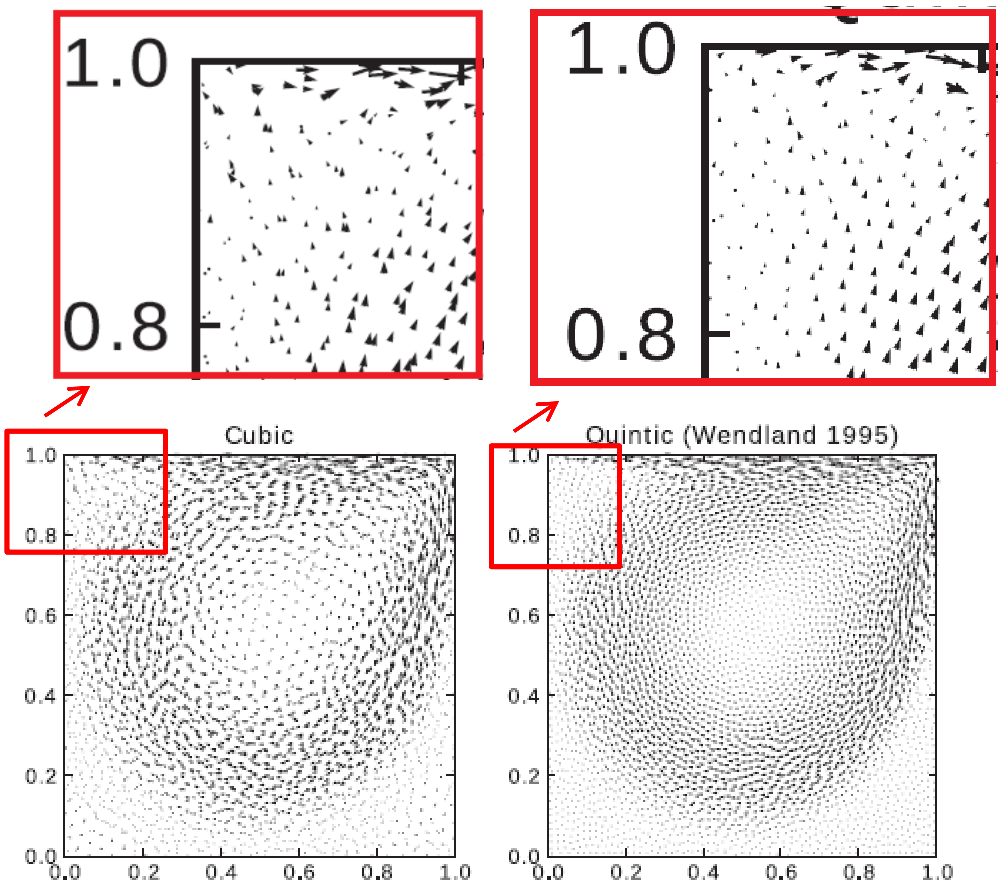
Choice of SPH kernel

properties

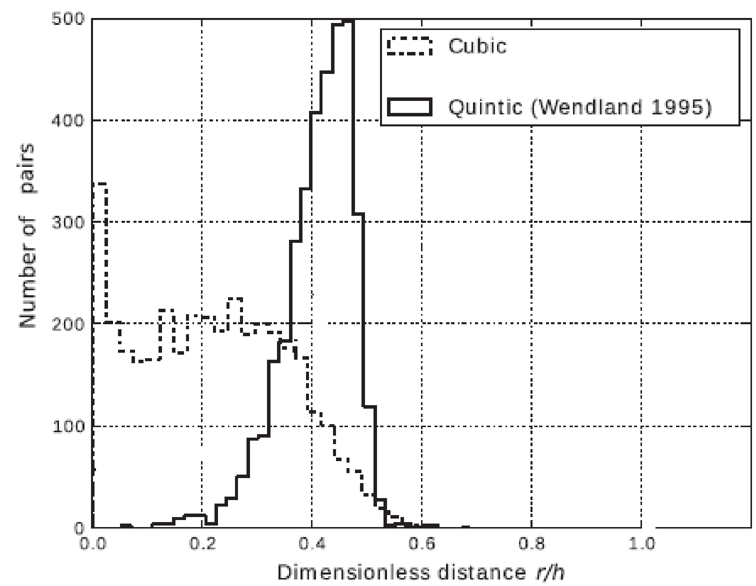
- some kernel functions may cause particle clustering (unwanted subkernel effect)
- computational efficiency is affected



[Szewc, Pozorski & Minier, IJNME **92** (2012) 343]



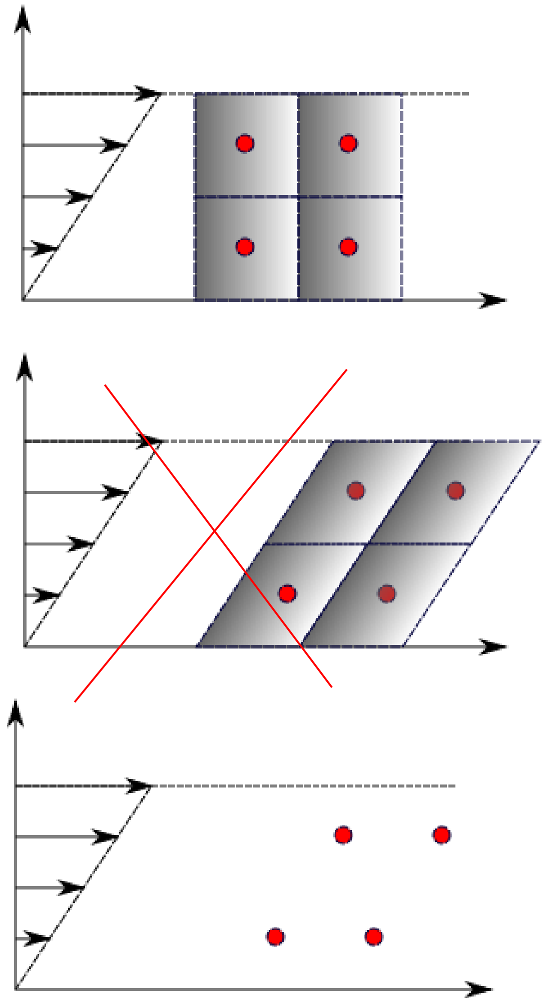
The lid-driven cavity flow: snapshots of particle locations.



The histogram of interparticle distance.

SPH particles vs. fluid elements

- discretisation of matter (fluid continuum) and not really of space (computational domain)
- the fluid element represented as material point of given mass but no explicit deformation
- for each computational (SPH) particle, its density, velocity, temperature, etc. are followed



Application areas:

- fluid mechanics, multiphase flows, hydroengineering, geophysics
- solid mechanics: large deformations, fracture
- FSI problems
- astrophysics

≠ "distortion of Lagrangian mesh"

? need of remeshing

$$\frac{d\mathbf{r}}{dt} = \mathbf{u}$$

$$\frac{d\varrho}{dt} = -\varrho \nabla \cdot \mathbf{u}$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\varrho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}$$

Notation: recall that

$$\langle A \rangle_a = \sum_b A_b W_{ab}(h)\Omega_b$$

$$\langle \nabla A \rangle_a = \sum_b A_b \nabla_a W_{ab}(h)\Omega_b$$

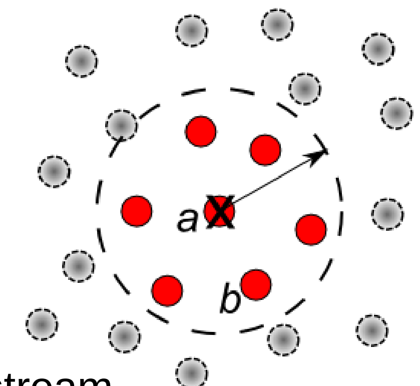
where

$$A_b = A(\mathbf{r}_b)$$

$$W_{ab} = W(\mathbf{r}_a - \mathbf{r}_b, h)$$

The particle advection equation

$$\frac{d\mathbf{r}_a}{dt} = \mathbf{u}_a$$



The SPH representation of mass conservation:

- the continuity equation

$$\frac{d\varrho_a}{dt} = -\varrho_a \sum_b \frac{m_b}{\varrho_b} \mathbf{u}_b \cdot \nabla_a W_{ab}(h)$$

to assure zero-order consistency, i.e., constant density in a uniform stream, another form is used

$$\frac{d\varrho_a}{dt} = \varrho_a \sum_b \mathbf{u}_{ab} \cdot \nabla_a W_{ab}(h)\Omega_b$$

$$\mathbf{u}_{ab} = \mathbf{u}_a - \mathbf{u}_b$$

- a direct summation

$$\varrho_a = \sum_b \varrho_b W_{ab}(h)\Omega_b = \sum_b m_b W_{ab}(h)$$

another form
(suitable for interfacial flows)

$$\varrho_a = m_a \sum_b W_{ab}(h) = m_a \Theta_a$$

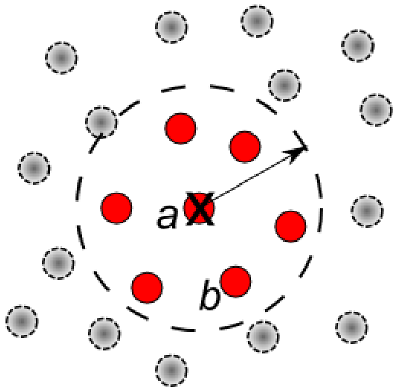
The Navier-Stokes equations:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{u}$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla\cdot\mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T - \tfrac{2}{3}(\nabla\cdot\mathbf{u})\mathbf{I}\right) + \mathbf{g} + \frac{1}{\rho}\mathbf{F}_s$$

Constant-property N-S eqs.:

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\varrho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{g}$$



The RHS terms of the momentum equation:

- the pressure term
  - simple discretisation

$$\left\langle\frac{1}{\varrho}\nabla p\right\rangle_a = \frac{1}{\varrho_a}\sum_b \frac{m_b}{\varrho_b} p_b \nabla_a W_{ab}(h)$$

  - momentum-conserving form

$$\left\langle\frac{\nabla p}{\varrho}\right\rangle_a = \frac{1}{m_a}\sum_b\left(\frac{p_a}{\Theta_a^2} + \frac{p_b}{\Theta_b^2}\right)\nabla_a W_{ab}(h)$$

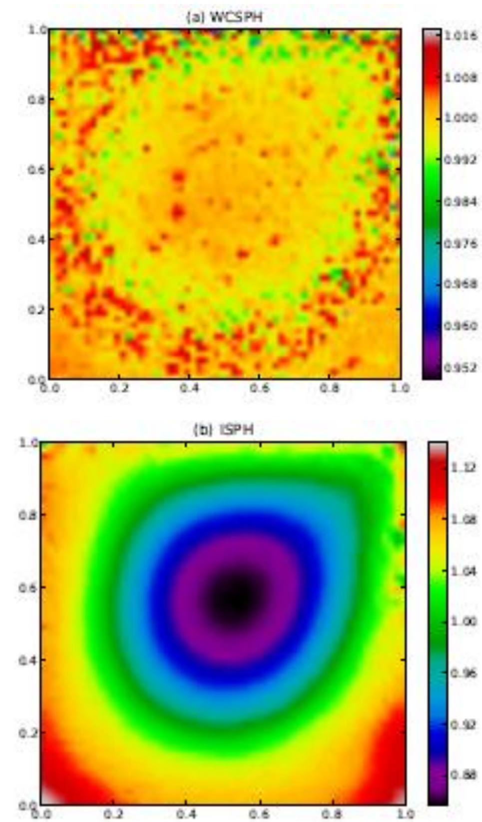
- the viscous term

$$\langle\nu\nabla^2\mathbf{u}\rangle = \frac{1}{m_a}\sum_b \frac{2\mu_a\mu_b}{\mu_a+\mu_b}\left(\frac{1}{\Theta_a^2} + \frac{1}{\Theta_b^2}\right)\frac{\mathbf{r}_{ab}\cdot\nabla_a W_{ab}(h)}{r_{ab}^2 + \eta^2}\mathbf{u}_{ab}$$

Finally, either

- the equation of state (WC SPH variant), or
- truly incompressible approach (ISPH)

SPH of incompressible flow



- weakly-compressible variant (WCSPH)

$$p = \frac{c^2 \varrho_0}{\gamma}\left[\left(\frac{\varrho}{\varrho_0}\right)^{\gamma} - 1\right]$$

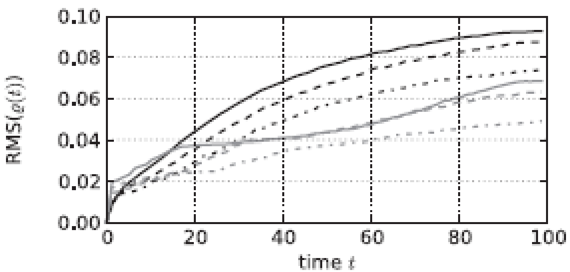
- truly-imcompresible variant (ISPH):
  - •• solenoidal velocity field [Cummins & Rudman, JCP 1999]

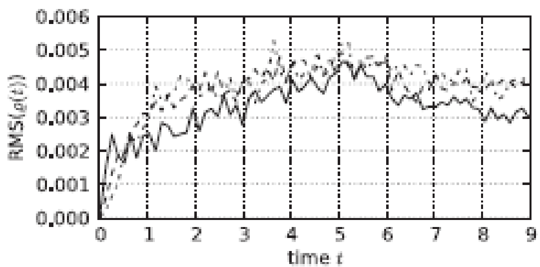
$$\nabla \cdot \left(\frac{1}{\varrho}\nabla p^{n+1}\right) = \frac{\nabla \cdot \mathbf{u}^*}{\delta t}$$

  - •• constant fluid density (double correction) [Pozorski & Wawreńczuk, JTAM 2002]

$$\frac{1}{\varrho_0}\nabla \cdot \left(\frac{\varrho^n}{\varrho_0}\nabla p^*\right) = 1 - \frac{\tilde{\varrho}^{n+1}}{\varrho_0}$$

↑ The lid-driven cavity flow: mean density field: WCSPH (upper map), ISPH1 (lower map). [Szewc 2013]



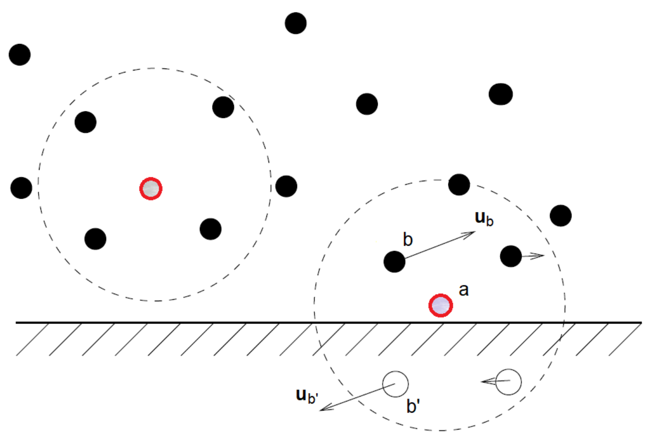


← Density r.m.s. in ISPH :
single correction (left graph)
double correction (right graph)

Boundary conditions in SPH

ghost particles, dummy particles

Ghost particles do not make part of the particle system (they are not followed in the simulation)

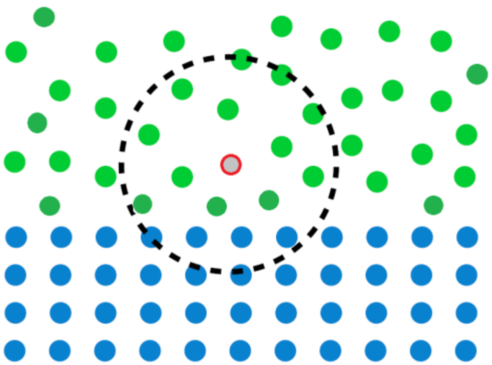


A sketch of ghost particles (empty circles)

$$\mathbf{u}_{a'} = 2\mathbf{u}_{\mathrm{w}} - \mathbf{u}_a, \quad m_{a'} = m_a,$$

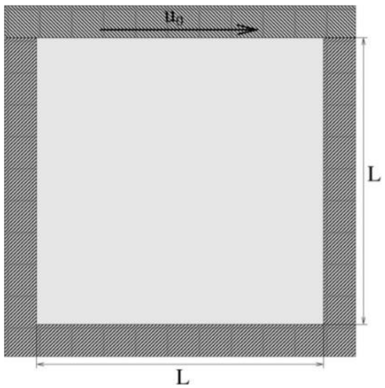
$$\varrho_a = \sum_b m_b W_{ab}(h).$$

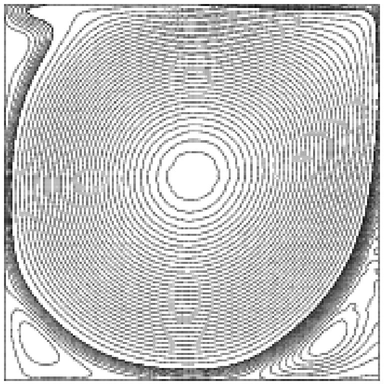
Dummy particles make part of the particle system, but their velocity is prescribed according to the BC

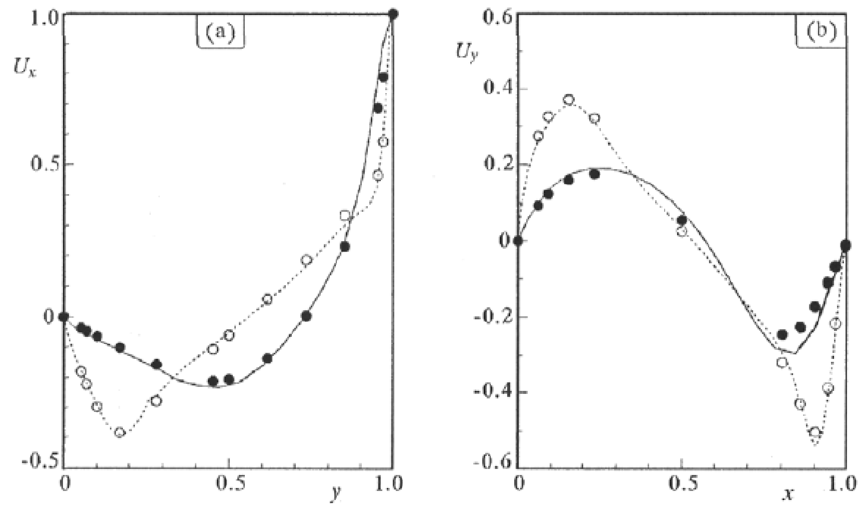


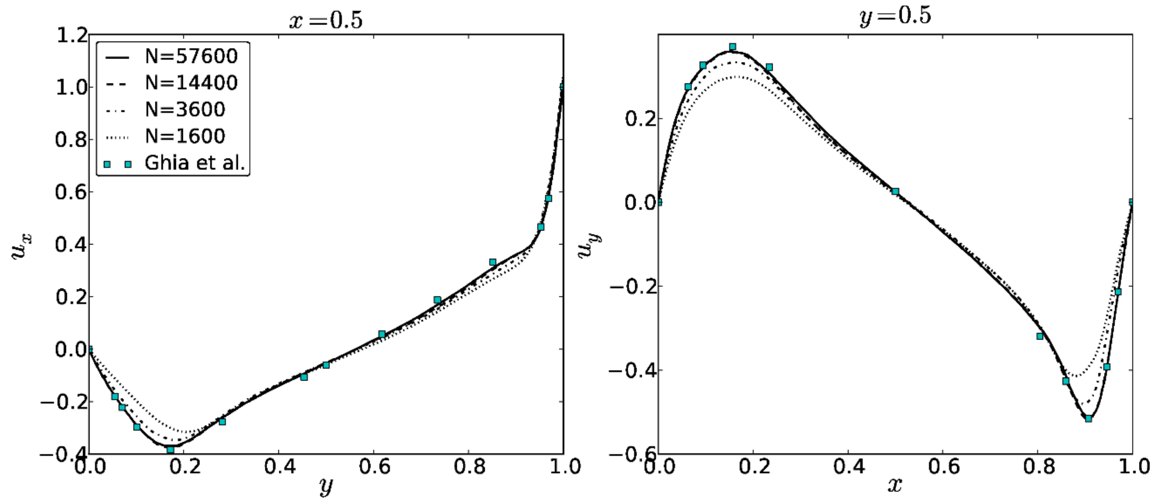
Smoothed Particle Hydrodynamics (SPH)

Validation case: lid-driven cavity flow



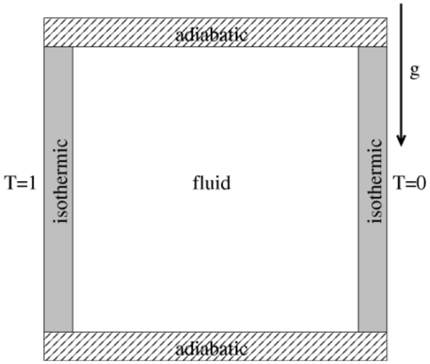


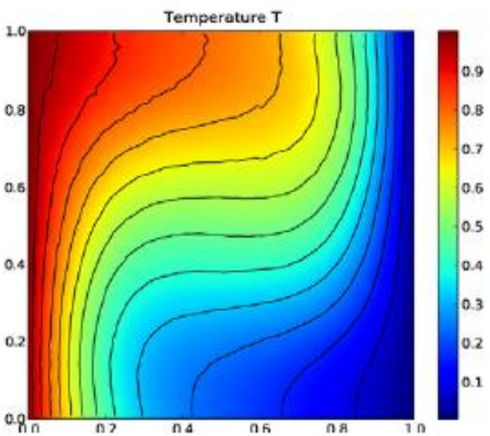




The lid-driven cavity, Re= 10^2 and 10^3 [Pozorski & Wawreńczuk, JTAM 2002]
ISPH with different number of particles, Re=10^3 [Szewc et al., IJNME 2012];
reference data: CFD computations [Ghia et. al., JCP, 1982].

SPH of differentially-heated cavity
beyond the Boussinesq approximantion

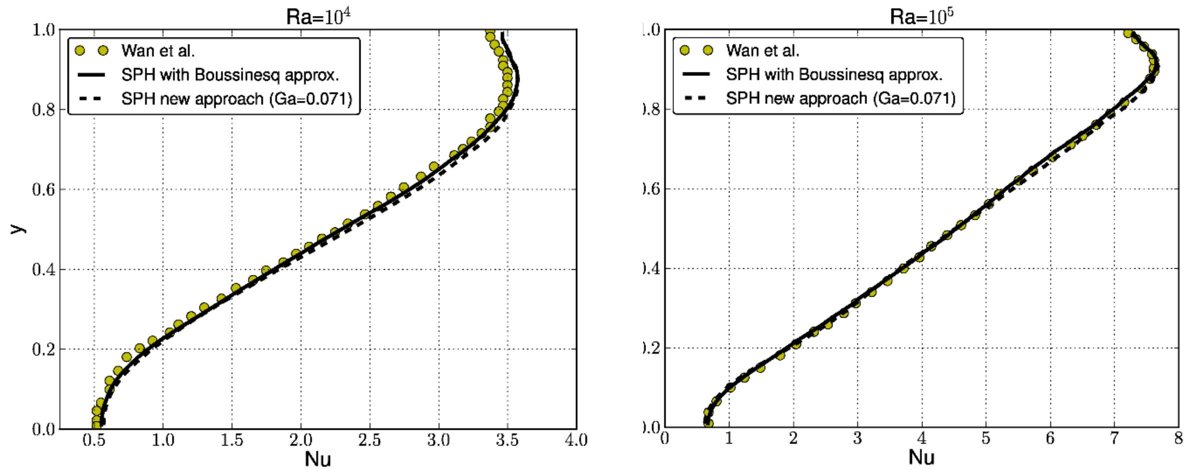




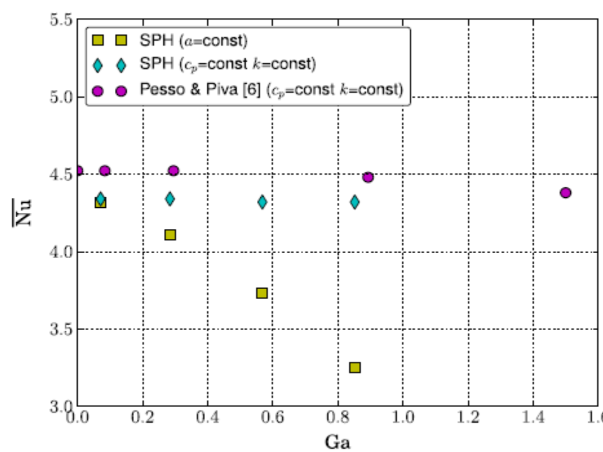
$$\frac{d\mathbf{u}}{dt} = -\frac{\nabla \delta p}{\varrho_0} + \nu \Delta \mathbf{u} - \beta \mathbf{g} \delta T$$

$$\varrho c_p \frac{dT}{dt} = \nabla \cdot (k \nabla T)$$

$$\frac{dT_a}{dt} = \frac{4}{\varrho_a c_{pa}} \sum_b \frac{k_a k_b}{k_a + k_b} \frac{T_{ab} \mathbf{r}_{ab}}{r_{ab}^2 + \eta^2} \cdot \nabla_a W_{ab} \Omega_b$$



Vertical Nu distribution:SPH with/without Boussinesq approx.
[ref. data: Wan et al., Num. Heat Transfer B, **40** (2001),199]



$Ga = \beta\Theta$

Heated cavity flow at Ra= 10^5, Pr =0.7, the effect of Ga
[Szewc, Pozorski & Tanière, IJHMT **54** (2011), 4807]

Flow modeling using SPH

Part II: Multiphase flows

- Mathematical models of separated and dispersed flows

  → surface tension, micromixing, two-fluid approach, delta-SPH

- Results:

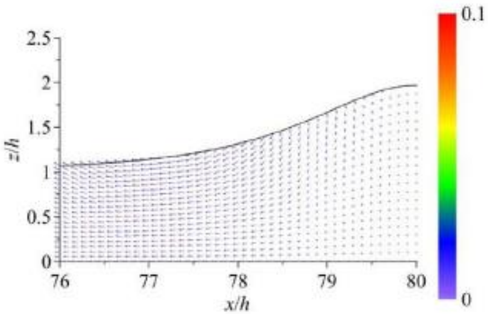
  → interfacial flows, wetting phenomena, sediment transport

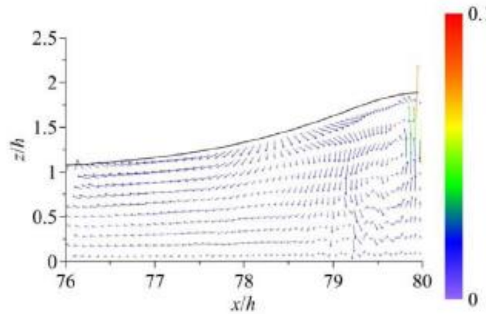
- Some major issues (Grand Challenges) in SPH:

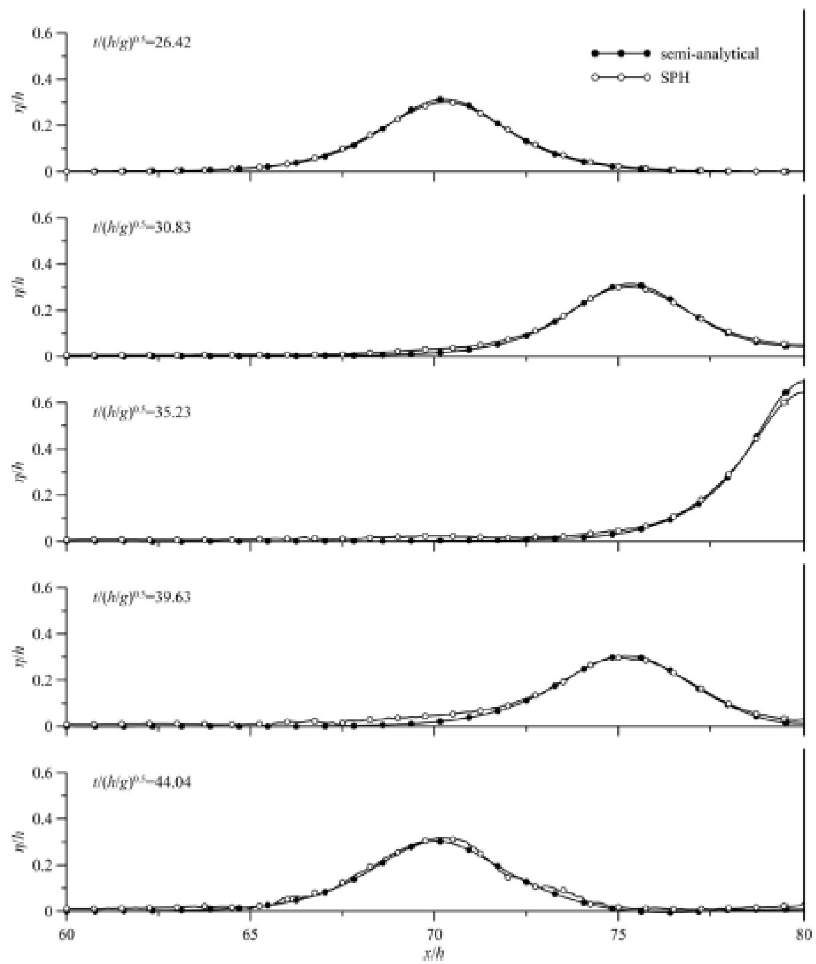
  → adaptivity, hybrid approaches

Free-surface flows

- no need for other fluid (air), unless dynamically important
- corrected kernel used (Shepard correction) to ensure consistency







Simulation of solitary wave impact on a seawall;
SPH compared to semi-analytical approach (SA).
Velocity field at maximum run-up: SA (up left) and SPH (up right);
Free surface evolution (sequence of graphs to the right).

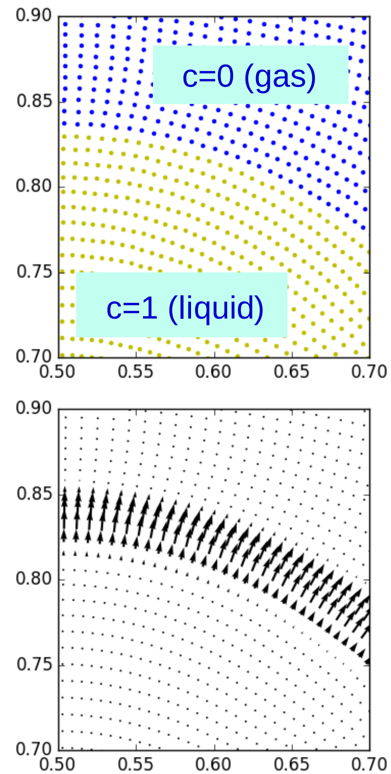
[Paprota, Staroszczyk & Sulisz; J. Hydro-Env. Res. 2018]

SPH representation of surface tension

surface tension at the interface represented as volume force ("continuum surface force" ,CSF) [Brackbill et al., JCP 1992]:

$$\mathbf{F}_s = \mathbf{f}_s \delta_s$$

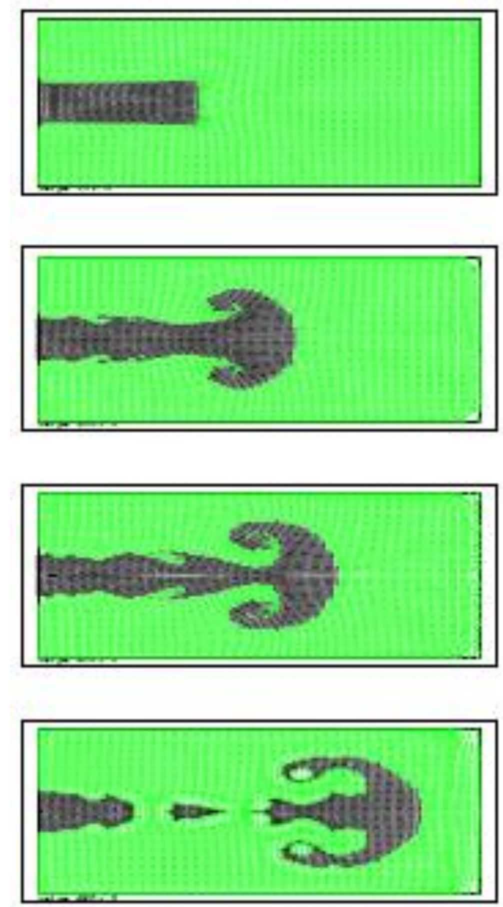
$$\mathbf{f}_s = \sigma \kappa \hat{\mathbf{n}} + \nabla_s \sigma$$



the normal vector based on phase indicator or "color function "c

$$\mathbf{n}_a = \sum_b c_b \nabla_a W_{ab}(h) \Omega_b$$

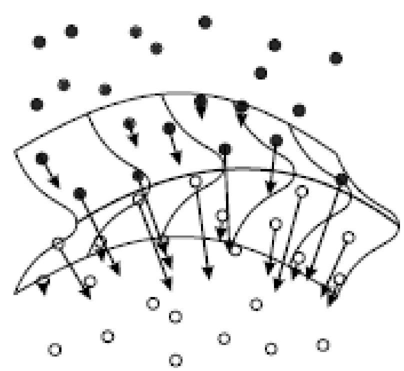
(in the simplest SPH setting)



SPH of liquid jet breakup [Wawreńczuk, 2004]

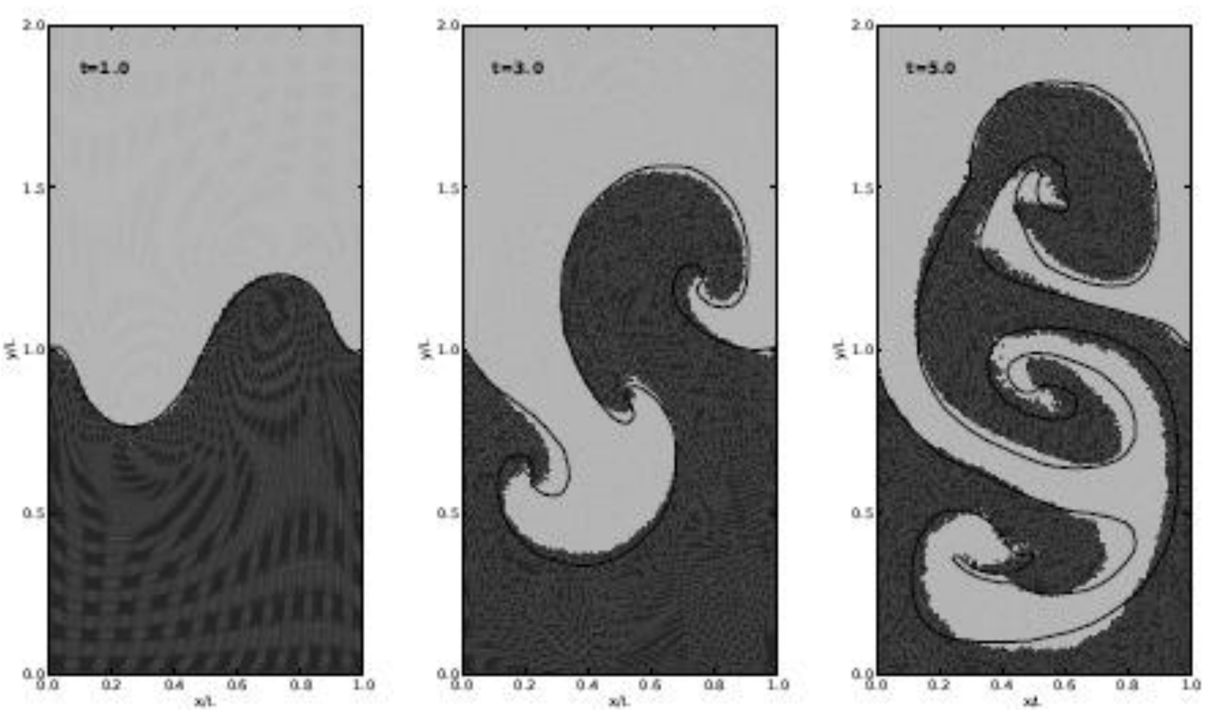
The unit normal and the interface curvature:

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{\nabla c}{|\nabla c|} \qquad \kappa = -\nabla \cdot \hat{\mathbf{n}}$$



Smoothed Particle Hydrodynamics (SPH)
The Rayleigh-Taylor instability

R-T: buoyancy-driven flow: control parameters: ρ_1/ρ_2 , v_1/v_2 , and Re= $(L^3g)^{1/2} / \nu$

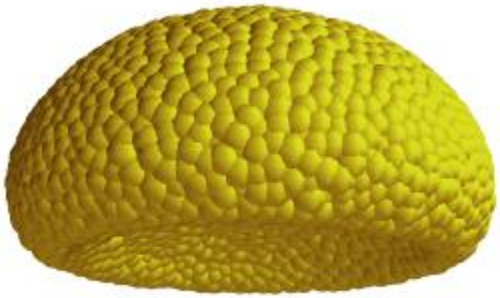


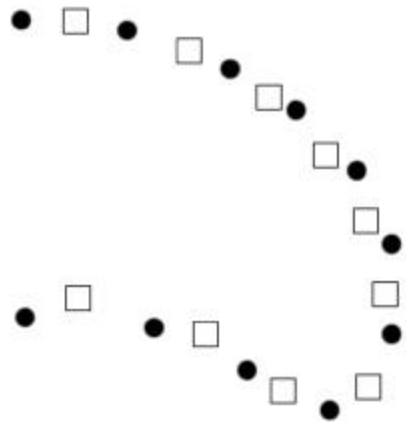
SPH results for R-T instability (128x256 particles) at Re=420

[reference data (lines): Level Set results of Grenier et. al., JCP **228** (2009), 8380]

Smoothed Particle Hydrodynamics (SPH)

rising bubble



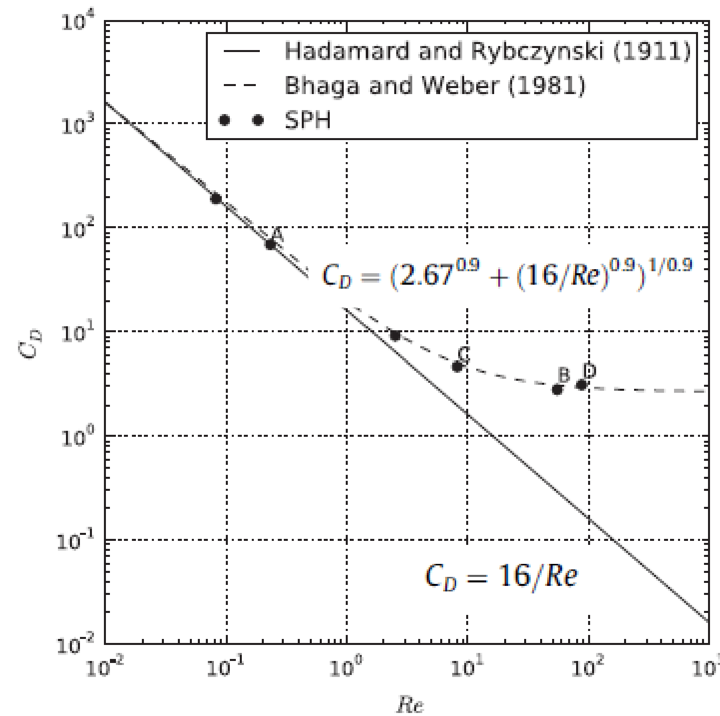


The control parameters are the Bond (Eötvös) and Morton numbers:

$$Bo = \frac{gD^2 \varrho}{\sigma}$$

$$Mo = \frac{g\mu^4}{\varrho\sigma^4}$$

SPH (•) vs. Front Tracking results (□) of Hua et al. (2008); single bubble of Bo=243 and Mo=266 rising through liquid
[Szewc , Pozorski & Minier., IJMF **50** (2013), 98]



Nearly-spherical bubble:
drag coeffcient.

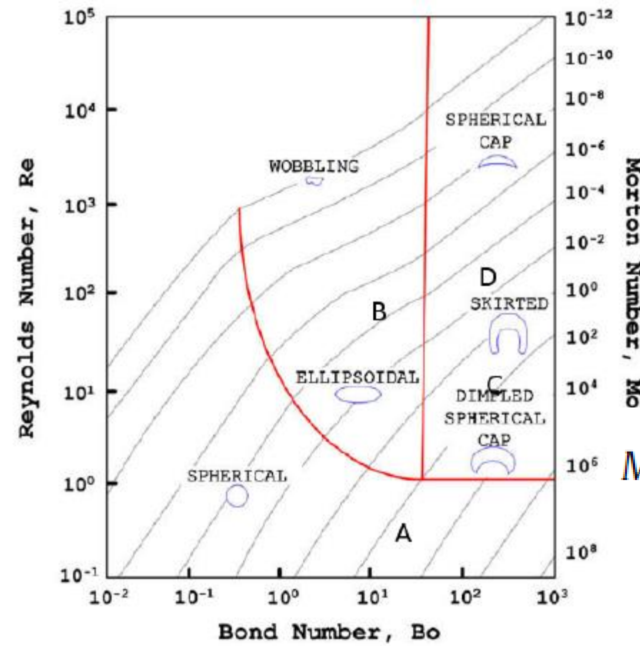
Smoothed Particle Hydrodynamics (SPH)
rising bubble (cntd.)

the experiment of Bhaga and Weber (1981)

Case	$E\ddot{o}$	Mo	Shape
A	17.7	711	oblate ellipsoidal
B	32.2	$8.2 \cdot 10^{-4}$	oblate ellipsoidal (disk)
C	243	266	oblate ellipsoidal (cap)
D	115	$4.63 \cdot 10^{-3}$	spherical cap

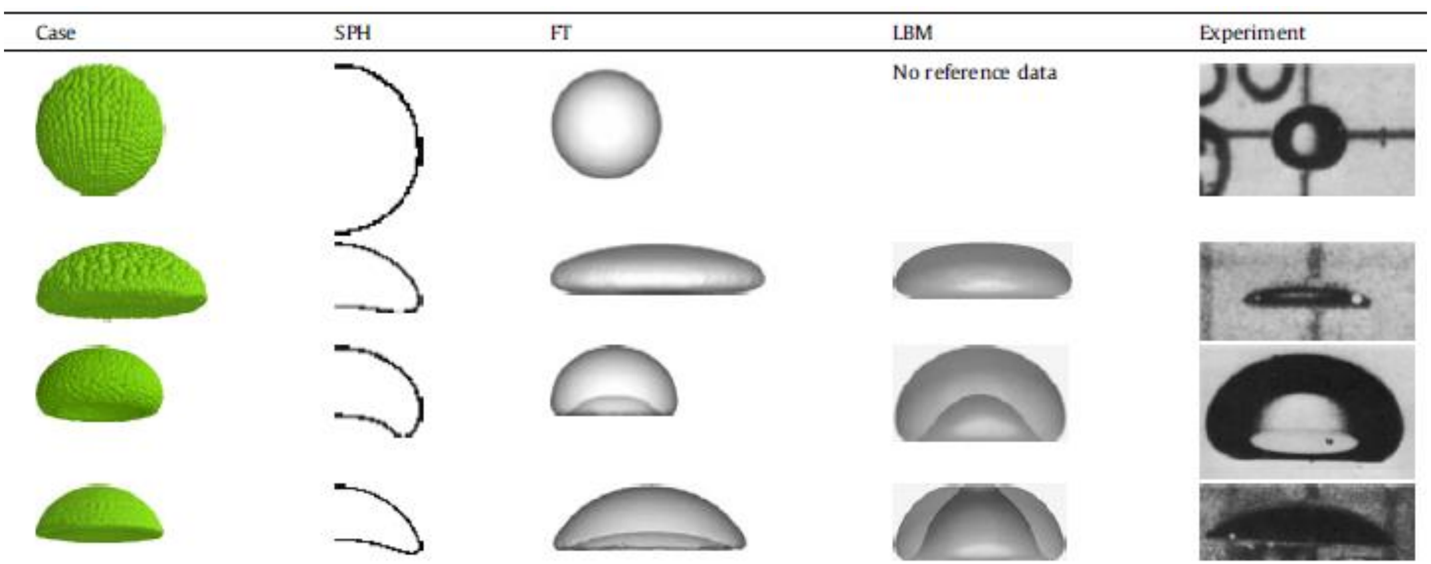
Map of bubble shape regimes →

[Amaya-Bower & Lee, C&F 2010]



$$Mo = \frac{g\mu_L^4}{\varrho_L \sigma^3}$$

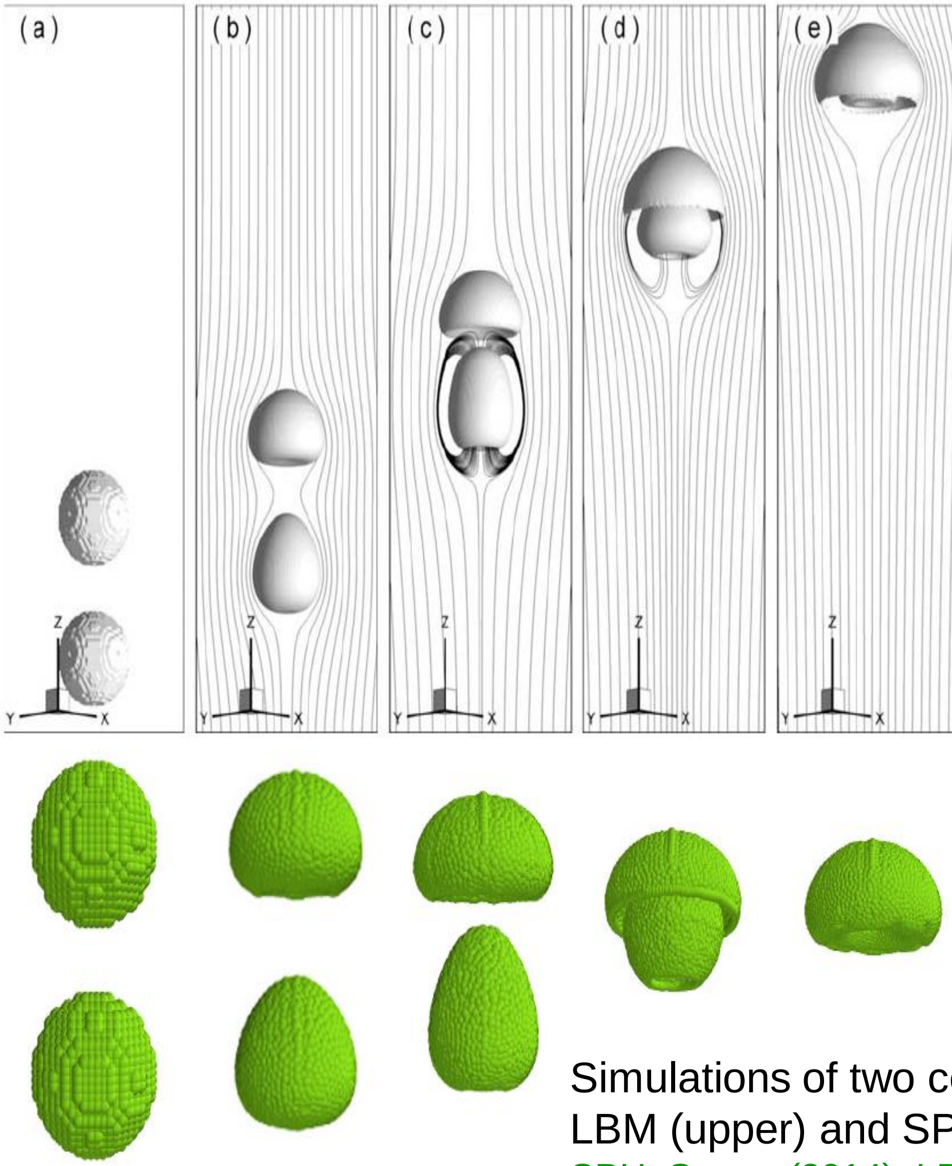
$$E\ddot{o} = \frac{gD^2 \varrho_L}{\sigma}$$



Shapes of bubbles rising through liquid

[Szewc et al., IJMF 2013]

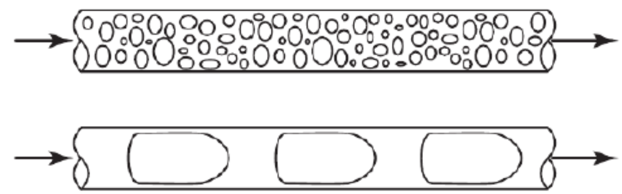
SPH of multiphase flow
two raising bubbles, "in line" setup



Simulations of two coalescing bubbles:
LBM (upper) and SPH (lower)
SPH: Szewc (2014), LBM: Cheng et al., C&F **39** (2010) 260]

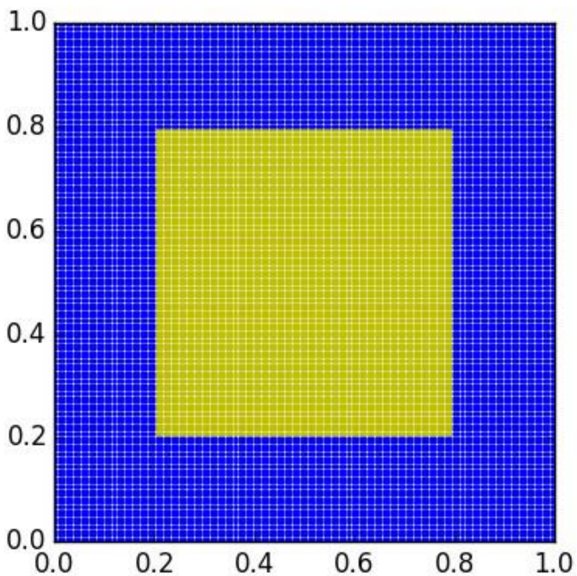
Interfacial area density; ”squircle”

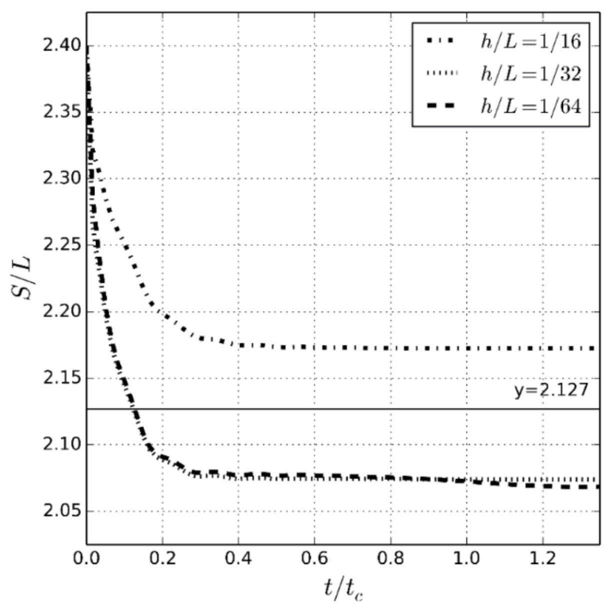
- a useful statistic in two-phase flows, related to the flow patterns
- in SPH**, the interface area** (in 3D) or length (in 2D) computed from

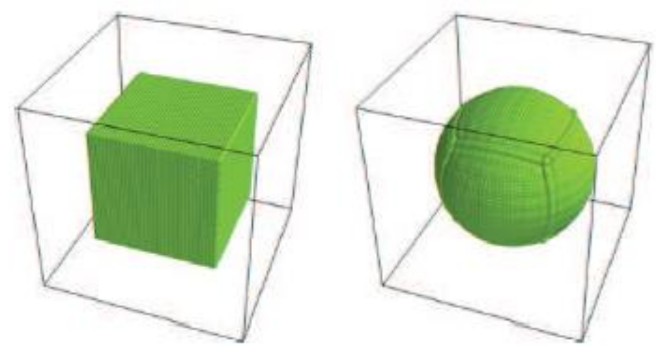


$$S = \int_\Omega \delta_S(\mathbf{x})\mathrm{d}\Omega\ , \quad S \approx \sum_a |\mathbf{n}_a| \frac{m_a}{\rho_a}$$

(δ taken as the normal vector length)

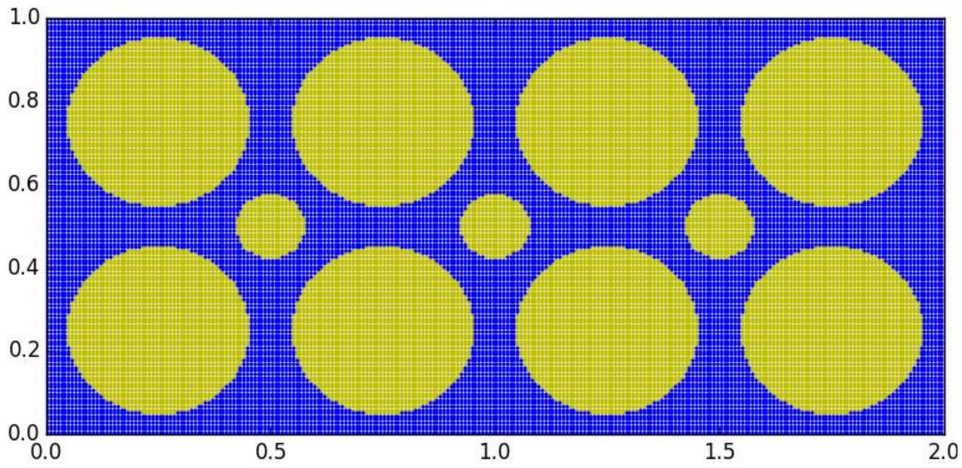




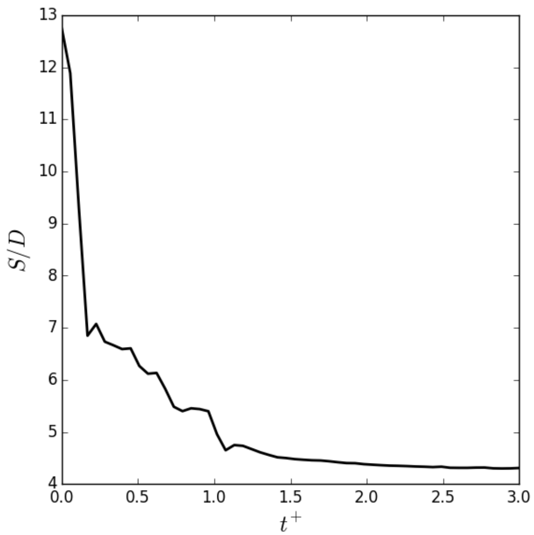


2D square-to-circle (squircle) deformation. Evolution of the interface length [Olejnik, PhD 2019]

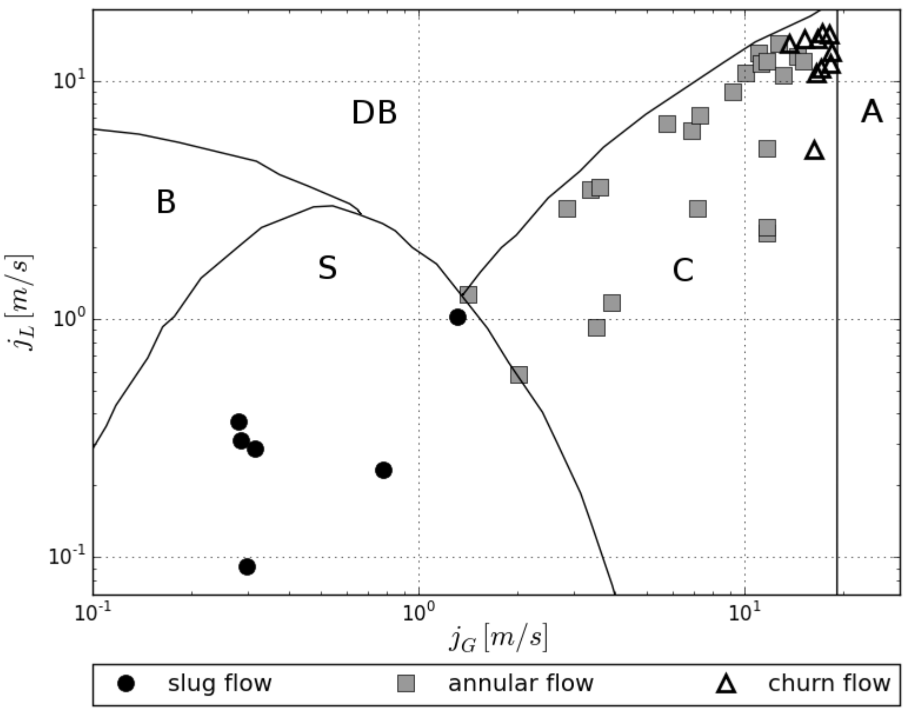
Two-phase flow in a channel:
slug flow; regime map



SPH simulations resulting in slug flow regime at $Re = 100$; gas volume fraction: 64%.

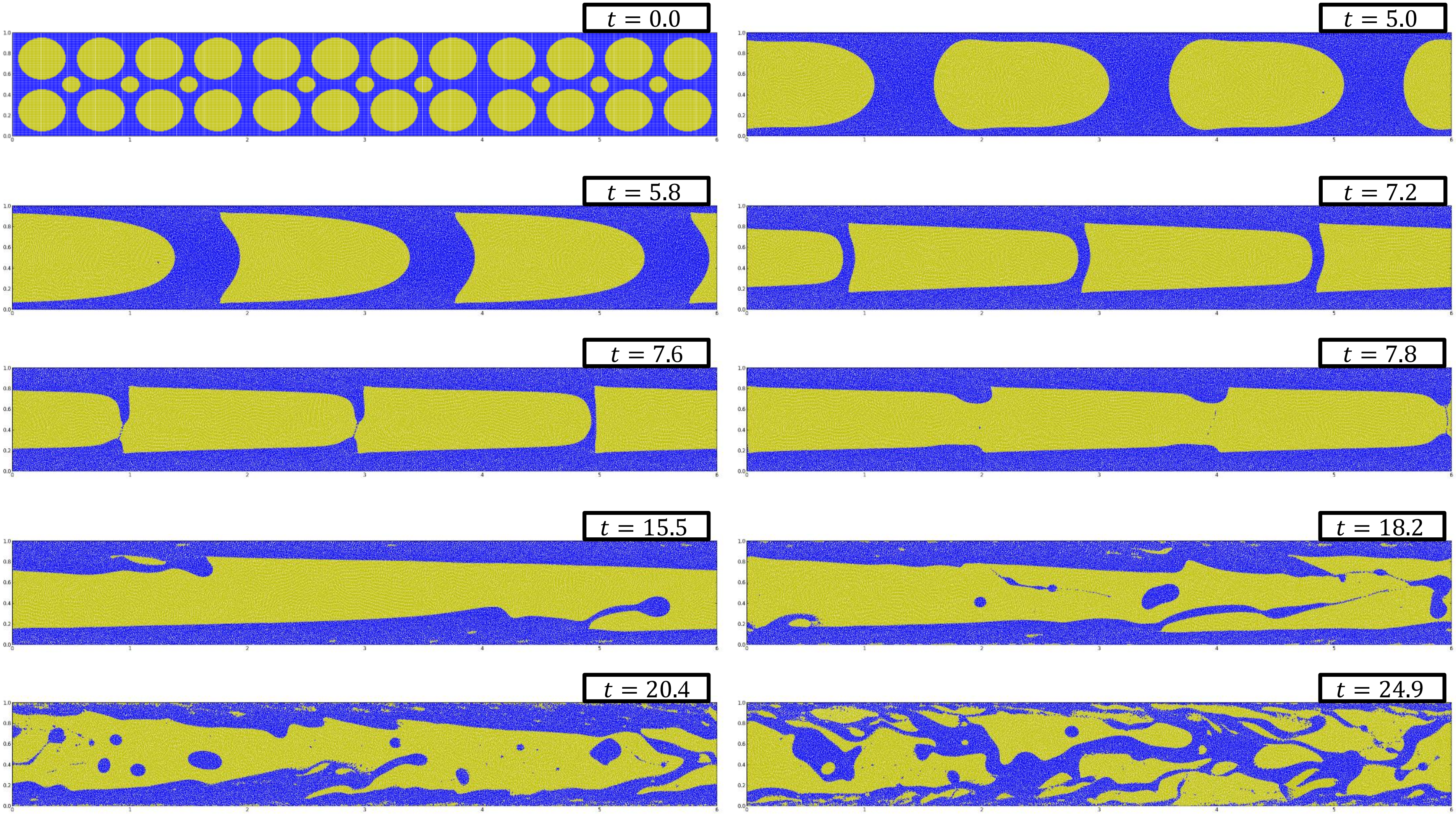


Evolution of the interface length



Flow regime map from SPH simulation; phases' superficial velocities coordinates.
lines & letters: generic flow map [Berna *et al.*, 2015]

Flow regime transition

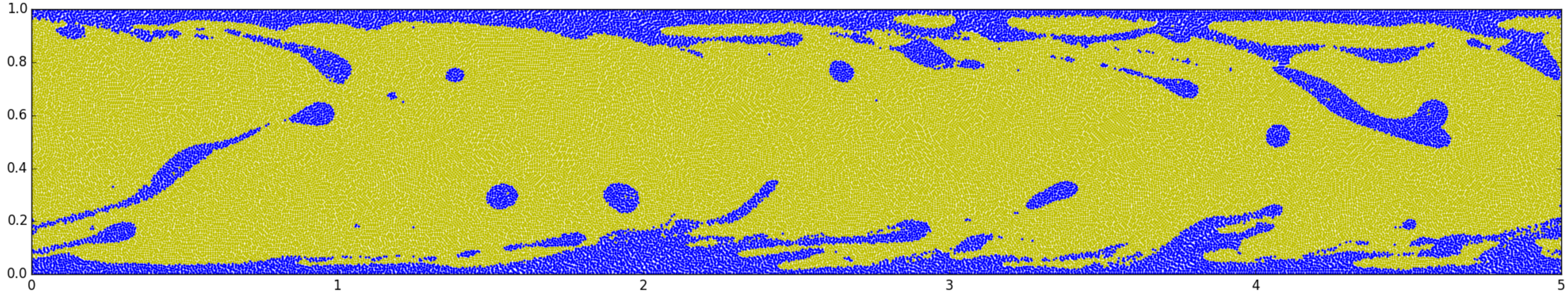


SPH simulation with increasing g (from $Re = 100$ to $Re = 3000$). [Olejnik, PhD 2019]

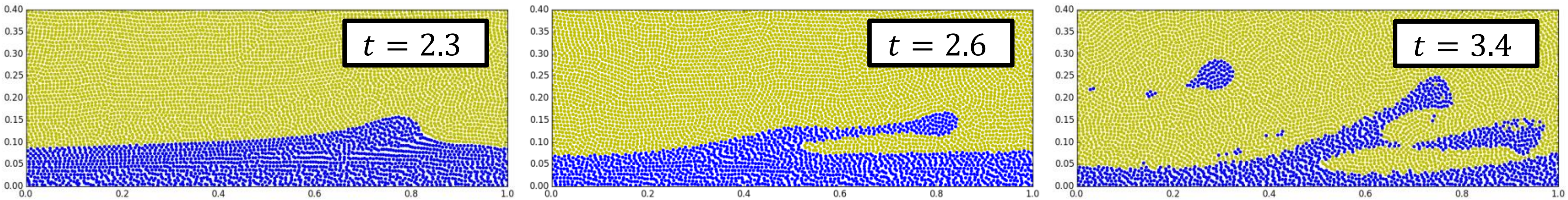
Annular flow

SPH simulation:

- 2D case (plane channel)
- liquid film dynamics, droplet separation and reentrainment



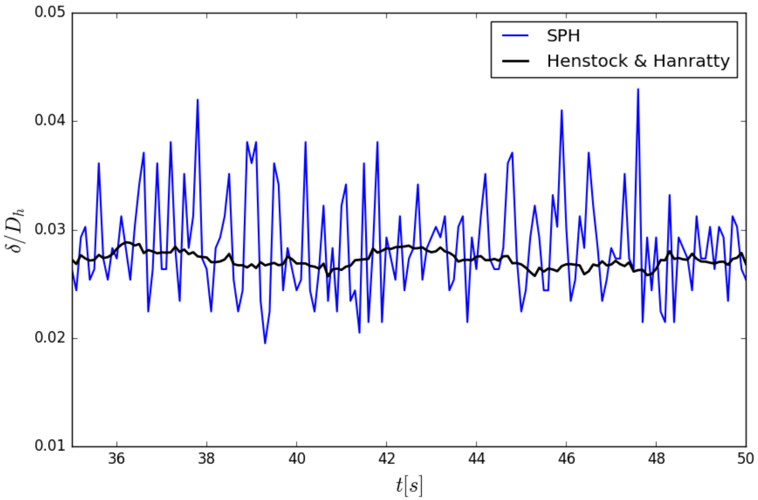
SPH simulation of annular flow: snapshot of the whole domain

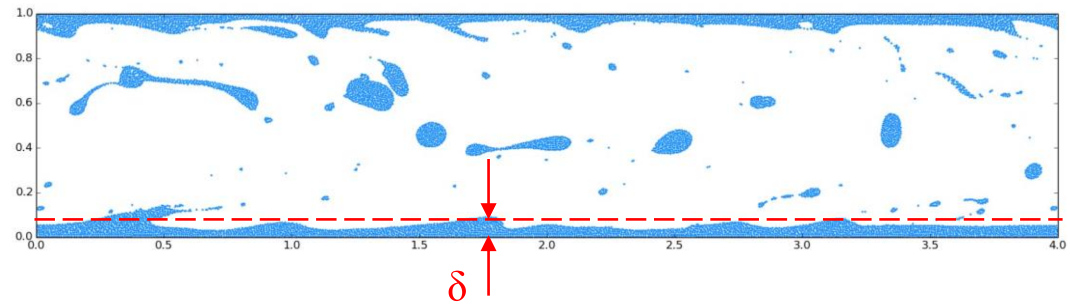


Zoom of the liquid film evolution

Annular flow (cntd.)

SPH estimation of film thickness [Olejnik, PhD 2019]





correlation for film thickness in vertical annular flow [Henstock and Hanratty, 1975]

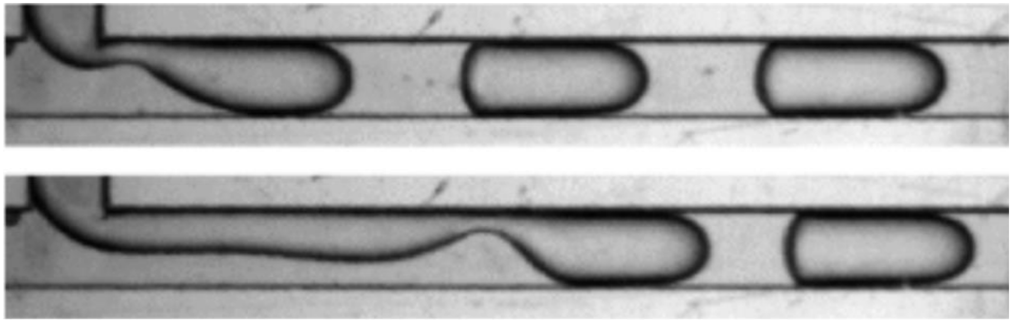
$$\frac{\delta}{D} = \frac{6.59F}{(1 + 1400F)^{0.5}},$$

where

$$F = \frac{1}{\sqrt{2}Re_G^{0.4}} \frac{Re_L^{0.5}}{Re_G^{0.9}} \frac{\mu_L}{\mu_G} \frac{\varrho_G^{0.5}}{\varrho_L^{0.5}}.$$

for SPH simulations in 2D channel, D is taken as hydraulic diameter $D_h = 2W$.

Wetting phenomena in SPH

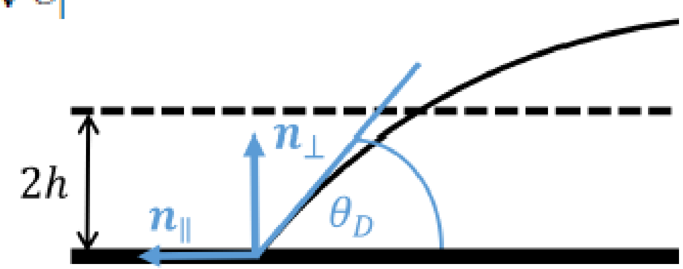


Generation of microfluidic droplets (lab-on-a-chip).
Korczyk *et al. J. Flow Chem.* (2015)

- improved contact angle model
- modified normal vector in the triple point region:

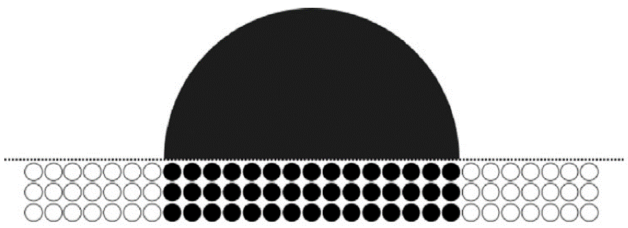
$$\mathbf{n}_{\mathrm{mod}} = \alpha \left(\mathbf{n}_{\perp} \cos\theta_D + \mathbf{n}_{\parallel} \sin\theta_D \right) + (1-\alpha) \frac{\nabla c}{|\nabla c|}$$

θ_D : contact angle,
$\alpha = \alpha(y)$: blending function

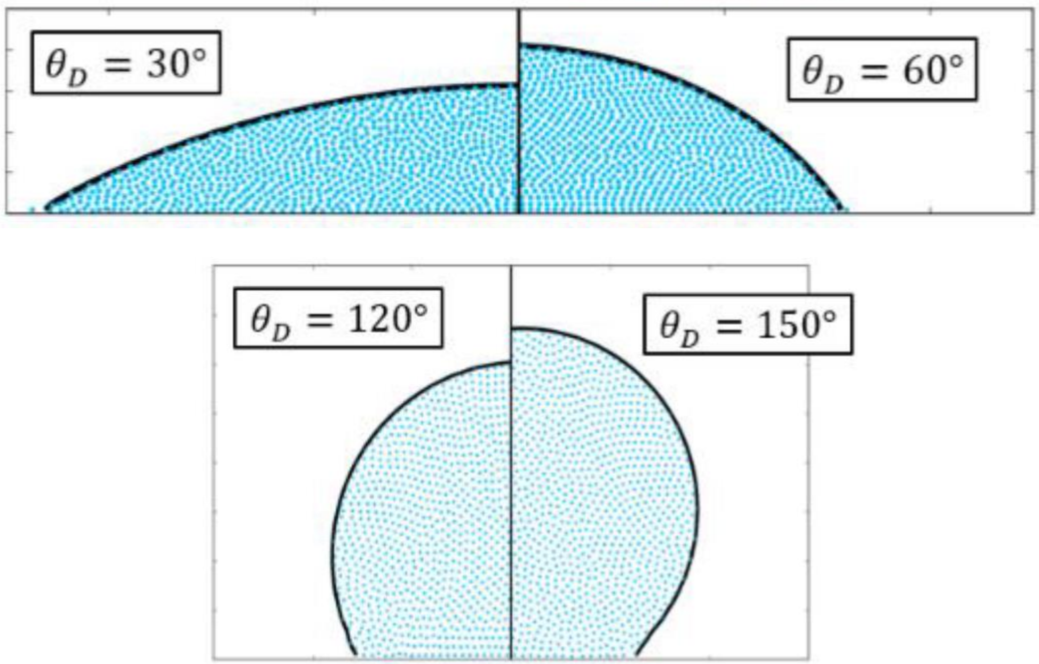


[Olejnik & Pozorski; Flow, Turbulence & Combustion 2020]

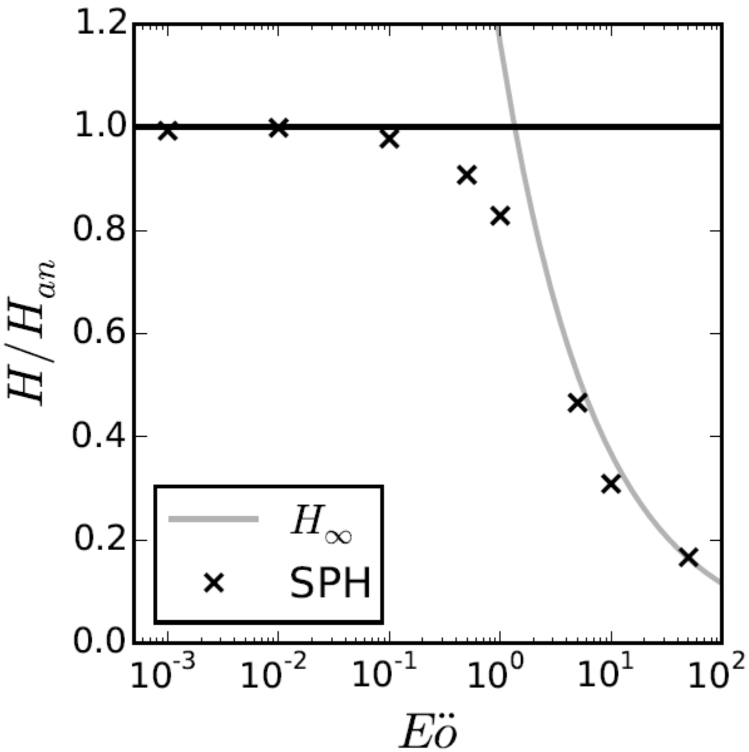
Wetting phenomena in SPH:
sessile droplets on hydrophilic/hydrophobic surfaces



Sessile droplet on a substrate (dummy particles used for wetting symulation)



Droplet shape: no gravity case

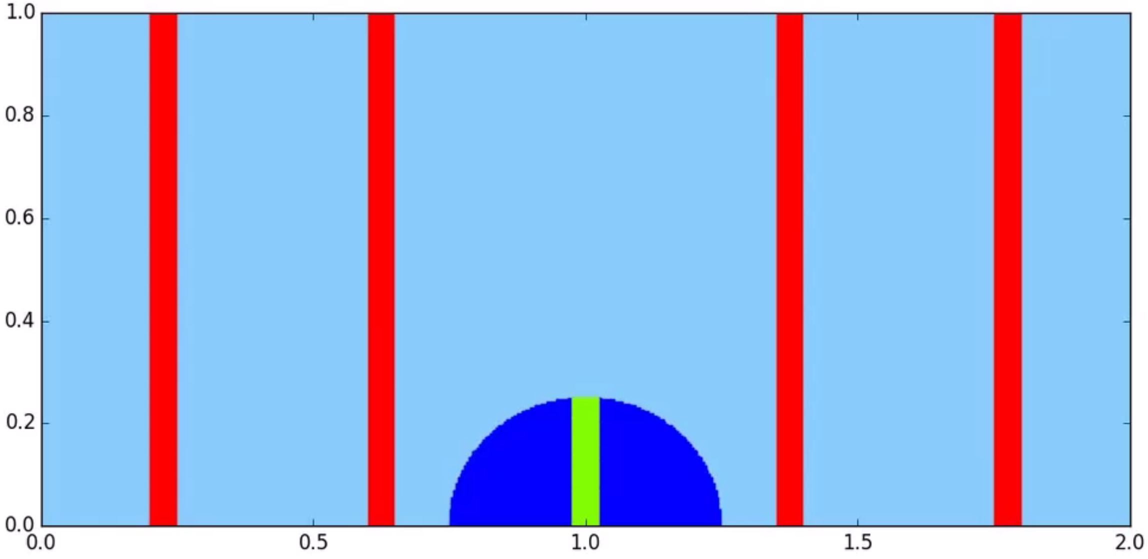


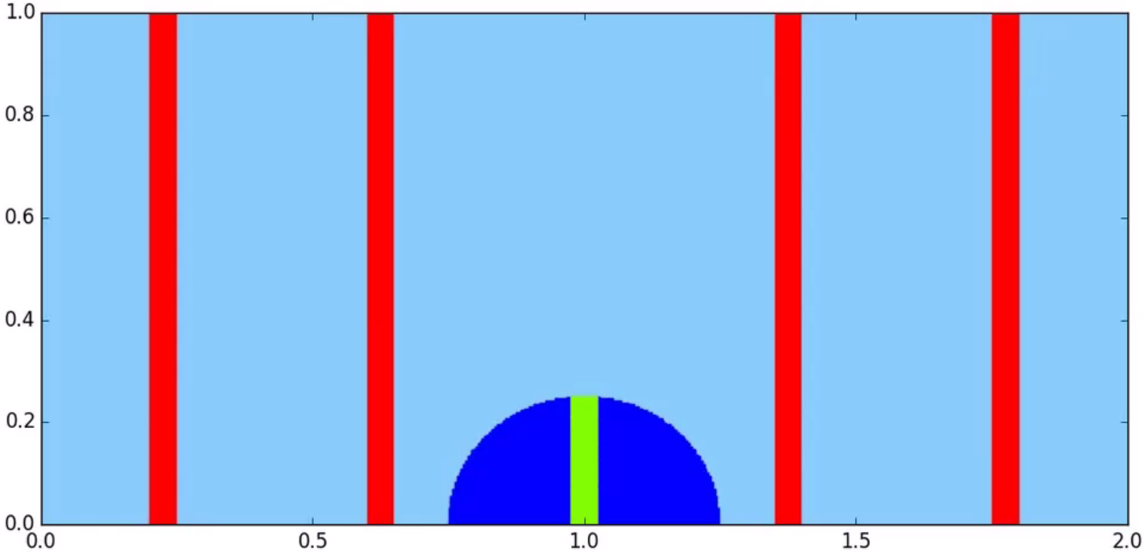
Sessile droplet shape:

comparison of the SPH results (symbols) with analytical solution (lines), 2D case. Surface tension and gravity dominated regimes for $\theta_D = 50°$.

Eötvös numer: $E\ddot{o} = \rho g d^2/\sigma$

Wetting phenomena in SPH (cntd.)

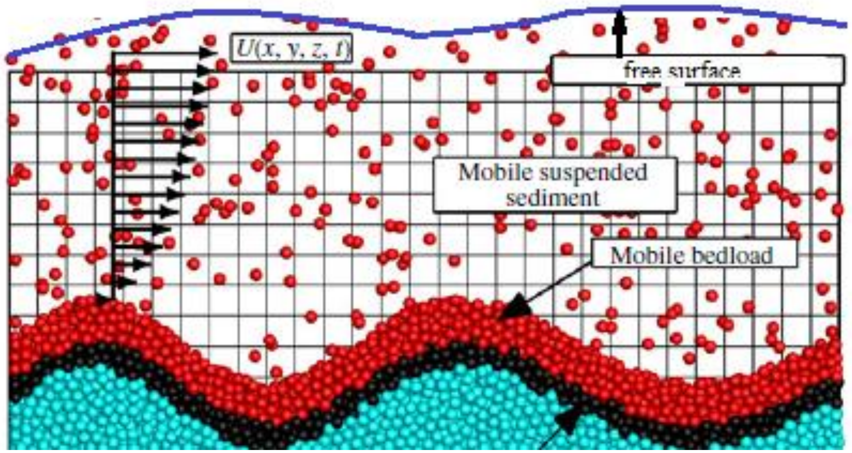




Air-flow driven motion of droplet on hydrophilic (θ_D=30°) and hydrophobic surfaces (θ_D=150°); capillary number Ca=1.

$$\varrho_L/\varrho_G = 1000, \mu_L/\mu_G = 100$$

Sediment dynamics
prospects for Smoothed Particle Hydrodynamics (SPH)



← free surface flow

← suspended sediment

← movable bed

Schematic of the problem;
adapted from [Finn, Li and Apte, JFM 2016]

Why SPH?

- a straightforward tracking of free-surface, no need of separate treatment for interface reconstruction
- some two-fluid models for two-phase flow in the bulk
- flexibility to incorporate complex physics, such as constitutive relationships for the sea-bed rheology

Two-fluid model of sediment transport

- interpenetrating continua (f-d or L-D)
- 2-W coupled momentum eqs.

$$\frac{d\mathbf{u}_f}{dt} = -\frac{\nabla p}{\rho_f} - \frac{K}{\hat{\rho}_f}(\mathbf{u}_f - \mathbf{u}_d) + \frac{1}{\rho_f}(\nabla\mu\cdot\nabla)\,\mathbf{u}_f + \mathbf{g}$$

$$\frac{d\mathbf{u}_d}{dt} = -\frac{\nabla p}{\rho_d} + \frac{K}{\hat{\rho}_d}(\mathbf{u}_f - \mathbf{u}_d) + \mathbf{g}$$

$$\frac{d\mathbf{x}_f}{dt} = \boldsymbol{u}_f$$

$$\frac{d\mathbf{x}_d}{dt} = \boldsymbol{u}_d$$

K=*K*(Re) is the interphase drag factor

- continuity eqs.:

$$\frac{d\hat{\varrho}_L}{dt} = -\hat{\varrho}_L\nabla\cdot\mathbf{u}_L$$

$$\frac{d\hat{\varrho}_D}{dt} = -\hat{\varrho}_D\nabla\cdot\mathbf{u}_D$$

where

$$\theta_L + \theta_D = 1$$

$$\hat{\varrho}_L = \theta_L\varrho_L$$

$$\hat{\varrho}_D = \theta_D\varrho_D$$

$$K = \begin{cases} 150\frac{\theta_D^2}{\theta_L^2}\frac{\mu}{d^2} + 1.75\frac{\theta_D}{\theta_L}\frac{\varrho_L}{\varrho_D}|\mathbf{u}_{LD}| & \text{when} \quad \theta_L < 0.8 \\ \frac{3\varrho_L\theta_L\theta_D C_D}{4d}|\mathbf{u}_{LD}|\theta_L^{-2.65} & \text{when} \quad \theta_L \geq 0.8 \end{cases}$$

[Gidaspow 1994]

the drag coefficient

$$C_D = \frac{24}{Re_p}\left(1 + \frac{3}{16}Re_p\right)^{0.5} \qquad Re_p = d|\mathbf{u}_{LD}|/\nu$$

Two-fluid model: SPH formulation

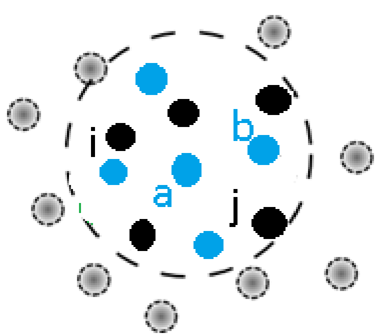
Continuity equations of two-fluid model [Kwon & Monaghan, IJMF 2015]

$$\frac{d\hat{\rho}_a}{dt} = -\hat{\rho}_a \sum_b \frac{m_b}{\hat{\rho}_b} \mathbf{u}_{ab} \nabla_a W_{ab}$$

$$\frac{d\hat{\rho}_i}{dt} = -\hat{\rho}_i \sum_j \frac{m_j}{\hat{\rho}_j} \mathbf{u}_{ij} \nabla_i W_{ij}$$

Two particle sets:

(a,b) for the fluid phase *f*
(i, j) for dispersed phase *d*



Two particle sets for the two phases

In SPH formalism

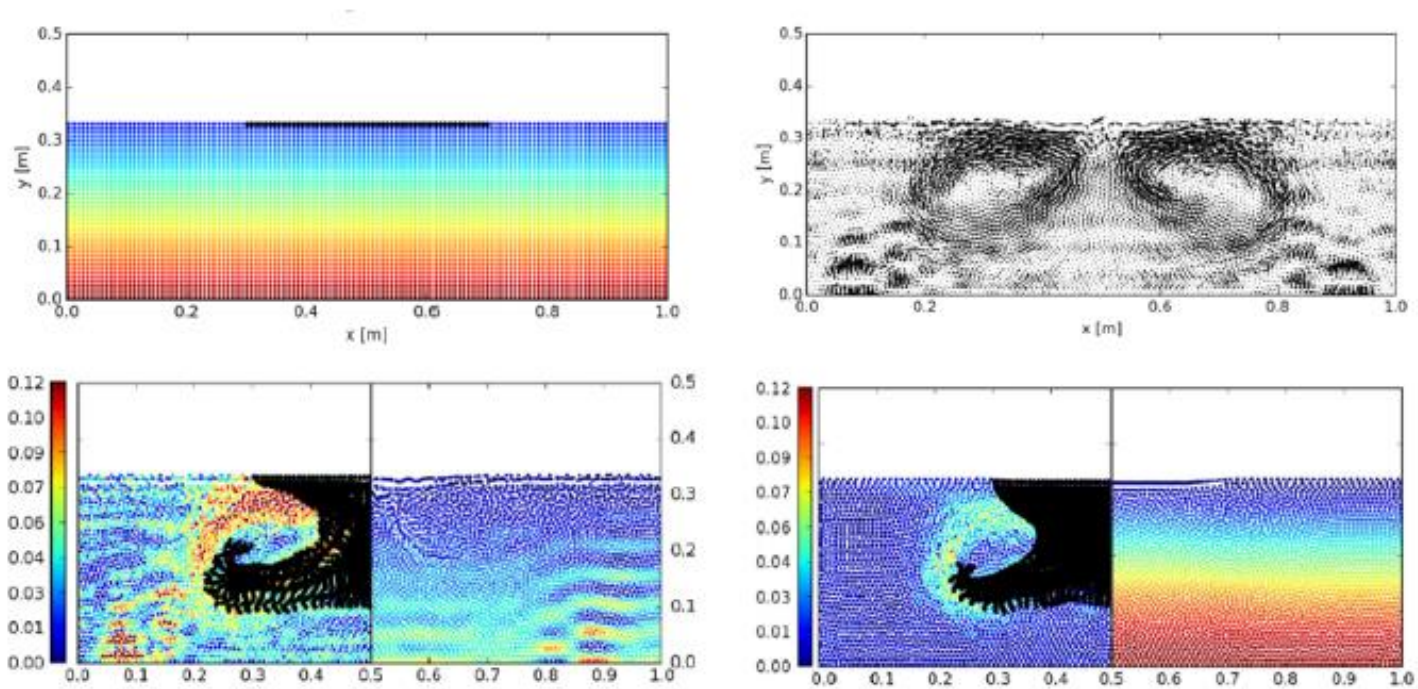
$$\frac{d\mathbf{u}_a}{dt} = \mathbf{g} + \sum_b m_b \left(\frac{\theta_a p_a + \theta_b p_b}{\hat{\rho}_a \hat{\rho}_b} + \Pi_{ab} \right) \nabla_a W_{ab} - \sum_j m_j \left(\frac{\theta_j p_a}{\hat{\rho}_a \hat{\rho}_j} \right) \nabla_a W_{aj} - D \sum_j m_j \frac{K_{aj}}{\hat{\rho}_a \hat{\rho}_j} \left(\mathbf{u}_{aj} \cdot \hat{\mathbf{r}}_{aj} \right) \hat{\mathbf{r}}_{aj} W_{aj}$$

Remedy for pressure oscillations: δ-SPH diffusive term

Add an extra term to the carrier phase continuity eq.
[Molteni & Colagrossi, Comp. Phys. Comm. 2009]

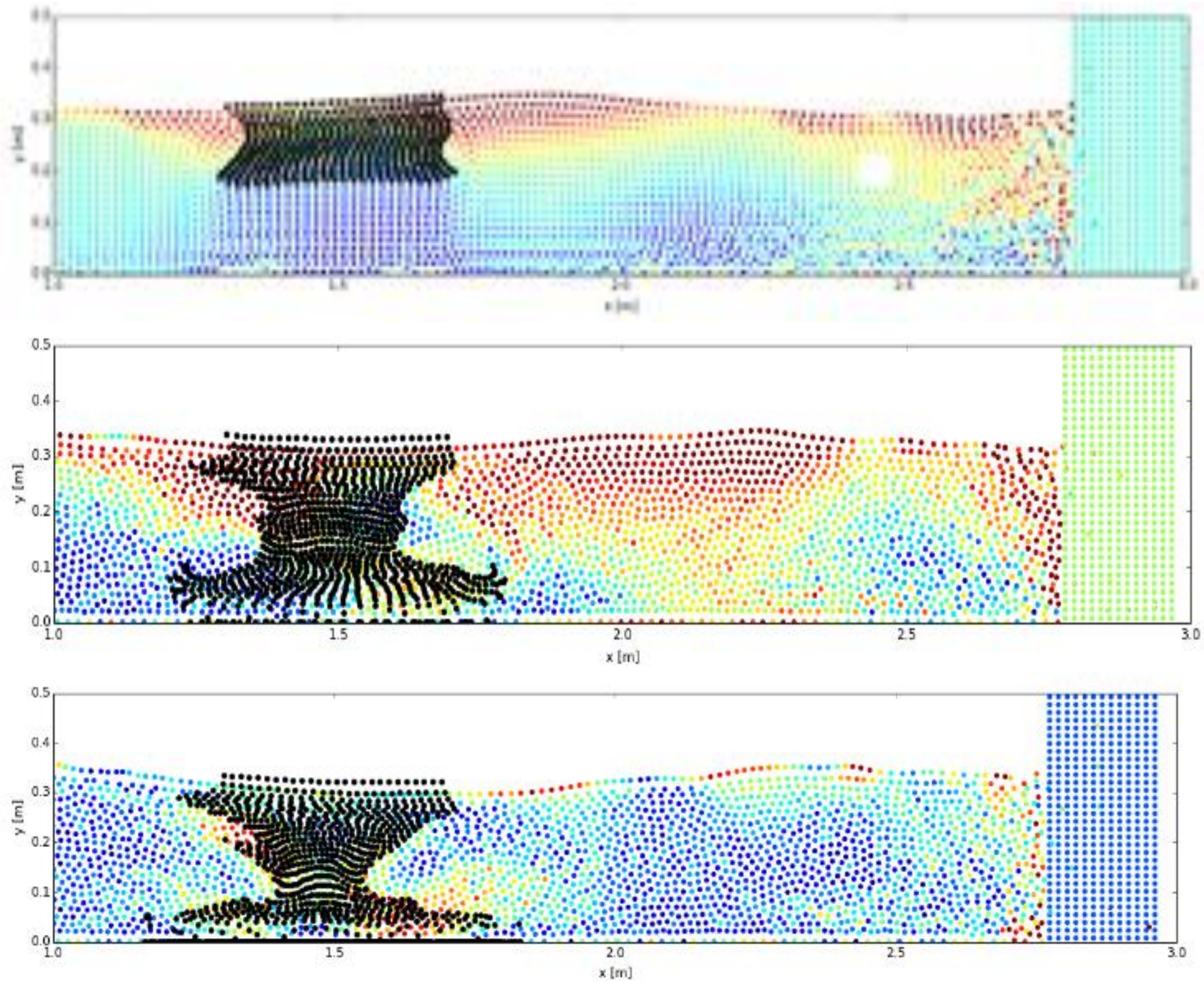
$$\frac{d\rho}{dt} = -\rho\nabla.\mathbf{u} + (\delta hc)\nabla^2 \rho$$

where δ≈0.1



SPH computation of sand dumping into water" Velocity magnitude and pressure oscillations: standard SPH (bottom left) with δ –correction (bottom right) [Olejnik et al., Int. Conf. PARTICLES 2017]

SPH simulations of flow in flume with wavemaker
continuous dumping of sand



Sediment transport and free-surface interaction using two-fluid SPH formulation.

SPH simulations of flow over movable bed

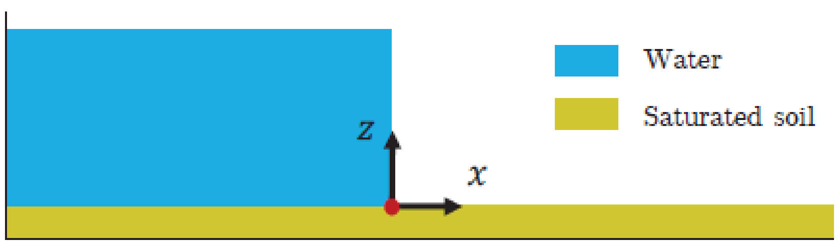
Equations of bed dynamics [Ulrich et al., Ocean Eng. 2013]

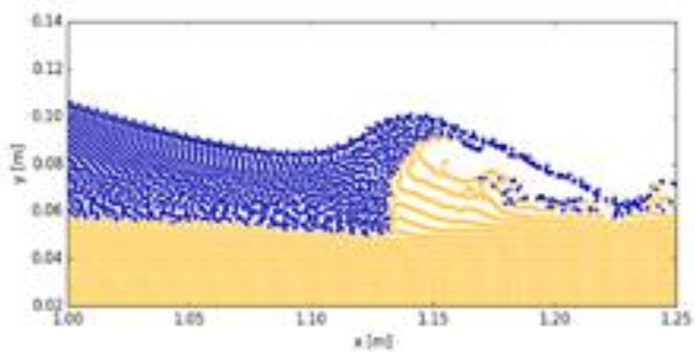
soil phase: viscous material of variable viscosity μ_s
(Mohr-Coulomb yield stress criterion)

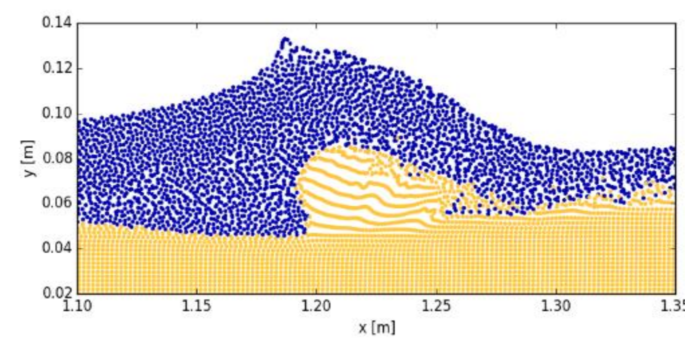
$$\mu_s = \min(\mu_s^*, \mu_{max}) \qquad \mu_s^* = \frac{\mathcal{C} + p \cdot \sin\Phi}{\sqrt{4\dot{\epsilon}^{\alpha\beta}\dot{\epsilon}^{\alpha\beta}}}$$

$$\mu_{max} \approx 1000 - 5000 \text{ Pa·s}$$

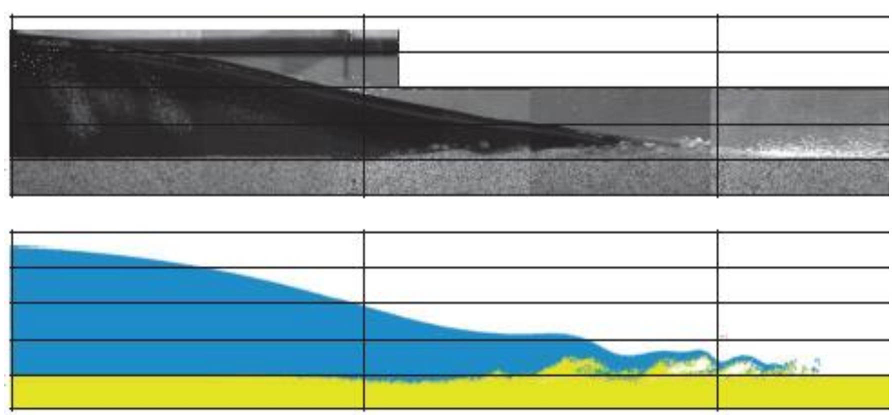
Φ is the internal friction angle
$\mathcal{C}$ is the cohesion parameter







Bed scour due to dam break
[Olejnik, 2018]



Dam-break wave on movable bed:
experiment and.SPH simulation
[Ghaitanellis et al., AWR 2018]

SPHERIC group and "Grand Challenges"

SPH European **R**esearch **I**nterest **C**ommunity
(http://spheric-sph.org)

ERCOFTAC Special Interest Group (SIG40)

Grand Challenges (GCs) as defined by the SPHERIC Steering Committee:

- GC#1: Convergence, consistency and stability
- GC#2: Boundary conditions
- GC#3: Adaptivity
- GC#4: Coupling to other models
- GC#5: Applicability to industry

Software (selected), community

Open-source SPH codes:

GPUSPH first implementation of WCSPH run entirely on GPU with CUDA
suited for multi-GPU computing (which is still rare)
INGV Italy, EDF & CNAM France, JHU USA and other partners
https://www.gpusph.org/

PySPH Python-based SPH framework (IIT Bombay, India)
Muta, Ramachandran & Negi (2020) *Comp. Phys. Comm* **255,** 107283
https://github.com/pypr/pysph

SPHinXsys multiphysics and multiresolution library
Zhang et el. (2021) *Comp. Phys. Comm* **267**

DualSPHysics being intensely developed for free-surface flow phenomena;
regular courses and workshops
Dominguez et al. (2022) *Comput. Part. Mech.* **9,** 867
https://dual.sphysics.org/

SPHERIC:

- international community of researchers and industrial users
- ERCOFTAC Special Interest Group #40; http://spheric-sph.org
- regular Workshops organised (upcoming: June 2023)

Background pattern: www.spheric-sph.org

General summary:
lessons learned on SPH

- a meshless, particle method; fast growing community
- increasingly applied to various industrial and environmental problems
- better suited for free-surface & multiphase flows with complex interfaces
- hybrid approaches appear such as SPH-DEM (particle^2), SPH-FEM (particle-mesh)
- fundamental issues persist: accuracy & convergence; adaptive resolution
- challenging for turbulent flows (fully 3D), especially wall-bounded
- in general, computationally expensive (multi-GPU?)

Acknowledgments

- PhD students:

Dr. Arkadiusz Wawreńczuk (2004), Dr. Kamil Szewc (2013),
Dr. Michał Olejnik (2019), Ms. Eleonora Spricigo (expected 2023),
for discussions and actual work on the original results presented here

- Prof. Ryszard Staroszczyk & Dr. Barbara Stachurska (IBW PAN, Gdańsk)
for collaboration on experimental aspects of sediment transport

- funding support:

EDF R & D, Chatou (France)

National Science Centre (NCN, Kraków, Poland):

project OPUS 2013/11/B/ST8/03818, project PRELUDIUM 2018/29/N/ST8/00267

UE projects: FP7 Euratom "Nugenia”, H2020 ITN-EID ”COMETE”

Selected bibliography on SPH

Textbooks and reviews

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invitation to Advanced Course at CISM* Udine, 11-15 Sept. 2023

*** International Centre for Mechanical Sciences**

ACADEMIC YEAR 2023
The Morton Gurtin Session
Centre International des Sciences Mécaniques
International Centre for Mechanical Sciences

LAGRANGIAN APPROACHES TO MULTIPHYSICS TWO-PHASE FLOWS

Advanced School
coordinated by

Christophe Henry
Inria Sophia Antipolis Méditerranée
Valbonne, France

Jacek Pozorski
IMP - Polish Academy of Sciences
Gdansk, Poland

Udine *September 11 - 15* 2023

INVITED LECTURERS

Pep Español - UNED, Madrid, Spain
4 lectures on:
Microscopic basis of hydrodynamics of two-phase fluids; The Dissipative Particle Dynamics and Smoothed Particle Dynamics models for the simulation of two-phase fluids.

Jochen Fröhlich - Technische Universität Dresden, Germany
6 lectures on:
Overview over numerical approaches for particle-resolving simulations; Euler-Lagrange methods with immersed boundaries for DNS of particle-laden and bubble-laden flows; Sub-scale models for collision, coalescence and breakup; Use of particle-resolving DNS for statistical modeling.

Christophe Henry - Inria Sophia Antipolis Méditerranée, Valbonne, France
6 lectures on:
Introduction to multiscale and multi-physics approaches; Common data model to couple modeling approaches with various levels of description; Hybrid macroscopic approaches for dispersed two-phase flows (mean-field/one-point pdf approaches); Application to environmental and industrial cases (e.g., plastic in rivers, fouling).

David Le Touzé - LHEAA, Ecole Centrale de Nantes, France
5 lectures on:
SPH method: theory, positioning with respect to classical CFD methods; Application to free-surface and (some) multiphase flows: physical assumptions, numerical models, examples of application; Coupling to other methods (FV for fluids, FE for fluid-structure interaction).

Alex Liberzon - Tel Aviv University, Israel
4 lectures + 2 hands-on sessions on:
Overview over the 2D and 3D Lagrangian particle tracking experimental methods from theoretical, computational, and hardware perspectives; Hands-on sessions on two-phase PIV and PTV methods for 2D/3D measurements, using open source software OpenPIV and OpenPTV.

Jacek Pozorski - Institute of Fluid Flow Machinery (IMP), Polish Academy of Sciences, Gdansk, Poland
5 lectures on:
Introduction to particle-fluid interactions in turbulent flows; Large-Eddy Simulation (LES) with point particles considering sub-scale effects (one-point stochastic and structural-type models); SPH modelling of interfacial flows.

Full info on:
cism.it/en/activities/courses/C2312